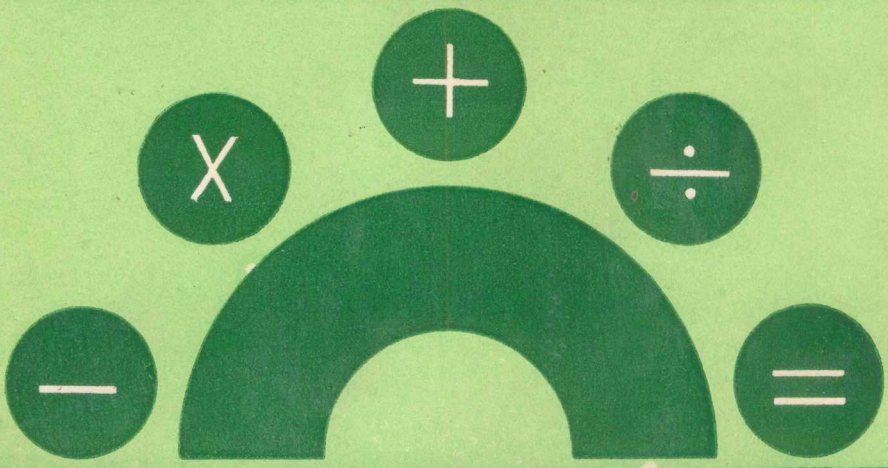
Mathematics Enjoyment For The Millions

J. N. KAPUR



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By
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PREFACE

Mathematics can be fun; it is certainly wonderful, exciting and fascinating. It can be enjoyed by every one.

Mathematics in fun can be earnest and mathematics in earnest can be fun. Mathematics is for the millions.

Mathematics is for those who love to climb intellectual mountains i.e., mathematics can be enjoyed by those who have some sense of intellectual adventure.

Mathematics has always been regarded as a tool for sharpening the brain and in the modern world of science and technology and computers, who would not like to sharpen his brain ? Won't it be wonderful if you can sharpen your brain and enjoy every moment of the process ? The present book is meant precisely to do this.

For sharpening the brain, one has to solve problems, special types of problems, problems which make you think systematically, logically and precisely.

Attempts at solving such problems will sharpen your brain, increase your thinking power and increase your power to reason systematically even in daily life.

Brahmgupta, the great Indian mathematician of the eighth century said, "If you want to shine in the company of the learned, propose mathematical problems and solve them."

Problem-solving and problem-posing are the heart and soul of the enjoyment of mathematics.

One need not be a mathematician to pose and solve mathematical problems. Every intelligent student and

every educated citizen who knows his middle school mathematics and who loves thinking and reasoning can do so.

Some persons enjoy solving cross-word puzzles, others enjoy playing chess. The former needs choice of proper words and the latter needs choice of proper moves Both need concentration and increase thinking power. Like cross-word puzzles and chess, mathematics is also an intellectual game, where you have to think of many alternatives and then use your reasoning power to choose one.

Mathematics is a superior game because each problem may require a different type of reasoning and every problem can be a challenge. It does not however require a superior intellect. Every body (and we mean everybody) can play this game of mathematics.

In Japan, almost all students and all adults play this game and they keep intellectually alert. In India also in ancient times, this game of problem-solving was played and then India was great, not only in mathematics, but in all fields of knowledge.

Let every Indian student and adult begin to play and enjoy this game of mathematics and then India will again be the intellectual leader of the world.

Let mathematics recreation clubs be formed in every school and every community centre and these will lead to an intellectual renaissance in India.

Let us use mathematical recreations for active enjoyment, for sharpening our brains and playing our parts better in today's science-based world.

In the present book, we have given fifty simple but intellectually stimulating problems. You can solve these problems in any order.

Only a few of these problems require any knowledge of mathematics beyond that of the high school.

These however require power of logical reasoning and concentration.

Mathematics is not a spectator sport. You cannot enjoy it by seeing orhers playing it. You can enjoy it by playing the game yourself; of course at your own level.

Mathematics is a game in wihch you play with numbers and geometrical shapes. Just as in other sciences, here you make thrilling and exciting discoveries. However unlike most other sciences, you can make these discoveries of beautiful patterns in numbers and geometrical forms, even if you do not have any physical equipment. Nevertheless your enjoyment of this game will be enhanced if you have a hand-held calculator at your disposal and it will be enhanced very significantly if you can use a computer, since in that case you can do numerical experiments on a large scale in very little time and you have a wonderful opportunity for making wonderful discoveries in number patterns. However for problems in this book, you do not need calculators or computers. We do hope that your enjoyment of problem solving here will encourage you to invent new games of your own to be played with calculators and computers.

In section A we have given fifty problems. Read each problem and spend at least ten to twenty minutes thinking about solving it with a paper and pencil in hand. It does not matter whether you can solve it or not. It is the mental struggle in attempting to solve it that is important. After this, turn to the solution of the problem given in section B. You may be surprised to find that the solution is simple. You may wonder why the solution did not occur to you in the first instance. The reason may be that you had allowed your reasoning power to rust. If you persist with these problems, your reasoning power should become sharper and you should find some subsequent problems easier to tackle.

After you have understood the solution of a problem, turn to the problem set at the end of the solution and try to solve the problems in the set. All these require the same type of reasoning power and you may be able to solve these. If not, discuss these with your friends.

We would propose that you pose problems of the same type on your own.

This book is meant for all those who love to THINK and to REASON. It will be useful for all those who want to learn mathematical habit of thinking, who want to do well in Talent Search examinations, mathematical olympiads, IIT and engineering college's admission tests etc. and who have the ambition to become first-rate mathematicians, scientists or technologists.

This book is also meant for those teachers and parents who want their students and wards to do well in mathematics, science and technology.

This book is also meant for all those adults in all walks of life who want active mental recreations, who want to make discoveries of their own, who love intellectual adventures and who can wonder at the great achievement of mathematicians in the domain of mathematics.

We do believe that the number of students and adults who can enjoy mathematics is in millions. We also believe that the number of persons who can enjoy mathematics, the most wonderful heritage of mankind, is also in millions.

With these remarks, we invite you to participate in the intellectual games in the pages ahead.

We hope you will love and enjoy these mathematical recreations, puzzles and games.

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J. N. Kapur

SECTION A

PROBLEMS

1. What was the last year in which there was no Friday the thirteenth ?
2. How many squares (of all sizes) are there on a chess board ?
3. What is the maximum number of regions into which a plane is divided by five lines, no two of which are parallel and no three of which intersect in a common point ?
4. If in a party everybody shakes hands with everybody else and there are 36 hand-shakes in all, how many persons are there in the party ?
5. In a regular polygon of seventeen sides, each vertex is joined to the other sixteen by a line which may be red, blue or green. Show that there will be at least one triangle with all its sides of the same colour.
6. Express all numbers from 1 to 50 as the sum of almost four square numbers.
7. Find a square number which is double of another square number.
8. One solution in integers of the equation $x^2+y^2+z^2=xyz$ is (3, 3, 6). Find ten more solutions in integers of this equation.
9. Take any three points on the circumference of a circle. Join these by straight lines. Show that you get four regions inside the circle. Similarly take

four and five points and show that you can get eight and sixteen regions respectively. Now take six points on the circumference of the circle and join them by straight lines. What is the largest number of regions that you can get ?

10. How many rectangles are there with perimeter 36 cms ? Which of these encloses the largest area and why ?
11. The digits of a certain integer are rearranged. Of all the numbers so obtained, the largest is divided by the smallest. Show that if the ratio is an integer, then it must be a square number.
12. A natural number (other than 1) which is divisible only by itself or unity is called a prime number. Thus 2, 3, 5, 7, 11, 13, 17..... are prime numbers. How many prime numbers are there in all ? How many prime numbers are less than 100 ? How many are less than 1000 ? Give ten consecutive numbers, none of which is a prime number.
13. A pair of prime numbers is said to be a twin prime pair if the two numbers differ by only 2. Thus (2, 5), (5, 7), (11, 13), are all twin prime pairs. How many twin prime pairs are there ? Verify that the digit sum of the product of two twin primes is always 8, except for the first pair viz. (3, 5). Also prove this result.
14. Show that 271 is the first prime factor of 13549729. Which is the next prime factor ?
15. If we rearrange digits, 123 can become 132, 213, 231, 312, or 321, we say there are six permutations of digits 1, 2, 3. Write all permutations of digits 1, 2, 3, 4, 5. If all the permutations of 1, 2, 3, 4, 5, 6 are written in increasing order, find the 42nd of these.

16. Find the next number of the sequence
11, 31, 71, 91, 32, 92, 13, 73, 14, 34, 74, 35, 95, 16, 76, 17, 37, 97, 38, 98

17. In how many ways can a square be divided into two congruent parts i.e. such that if one part is put over the other, these will fit completely.

18. Which capital letters of the English alphabet can be written in terms of straight line segments only and which numbers can be written in terms of these letters ? In which of these numbers is the number of straight line segments equal to the number itself ?

19. Find the smallest number which when divided by some number m from 1 to 20 always leaves m^{-1} as remainder.

20. Find a sequence of twelve numbers in which no number is divisible by any number and in which the first and last numbers are as small as possible.

21. A runner notes that wherever he is on the track, his distances from the two flags in the field one km apart, are in the ratio 2 : 1. How long is the track ?

22. At what temperature will the number of degrees Fahrenheit be the same as the number of degrees Celsius ?

23. Which is larger in area : a square of 9" side or a circle of diameter 10" ?

24. Find all the primes less than 1000 which are palindromes i.e. which read the same forwards and backwards ?

25. For how many values of n is the number
$2^{11}+2^{8}+2^{n}$ a perfect square ?

26. Find a Pythagorean triplet ($a^2+b^2=c^2$; a, b, c integers), in which all the three a, b, c are odd numbers.

27. The smallest number which can be expressed as the sum of two cubes in two different ways is the Hardy-Ramanujan number $1729=10^3+9^3=12^3+1^3$. Find the next four larger numbers with the same property.

28. In the space-age magic square

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

the sum of the elements of every row, column, diagonal is 34. The sum of the four corner elements and the sum of the four central elements is also 34. The sum of the four elements in each of the four blocks into which it is divided by the two central lines is also 34. There are in all 86 ways in which the sum of four elements situated at the vertices of a regular figure (square, rectangle, parellelogram y-figure etc.) is 34. Find at least forty of these. If you can find all the 86. write to the publisher who will send a free copy of the Mathematical Enjoyment II to the first five persons who independently send all the 86 solutions.

29. In the above magic square, there are patterns in sums of squares and cubes of elements. Find as many of these as possible.

30. Take any three digit number, rearrange the digits and subtract the smallest number from the largest number. Repeat the process. What pattern do you

observe? Find similar patterns for 4 and 5 digit numbers.

31. Consider the so-called Lucas numbers 1, 3, 4, 7, 11, 18, 29, 47.....in which any number after the second is the sum of the two preceding numbers. Prove that $L_n = 8\,L_{n-5} + 5L_{n-6}$ and deduce that not a single number of this sequence is divisible by 5.
32. Find the unit digit of $(3157)^{1024}$
33. I have one 'laddoo' of 2" radius. With the same material, how many laddoos of 1" radius can I make ?
34. Find the first value of n for which n^2+n+41 is not prime.
35. The integers 1, 3, 8 and n have the property that the product of any two when added to one yields a perfect square. What is n ?
36. Show that the sum of the areas of circles drawn with the sides of a right-angled triangle as diameters is equal to the area of the circle with the hypotenuse as diameter.
37. Solve USA+USSR=PEACE i.e. replace the letters of the alphabet by digits so that this becomes a true equation.
38. Find at least two values of n such that the sum of squares of first n numbers is a perfect square.
39. A perfect digital invariant is an integer N of k digits in which the sum of the nth powers of its digits is equal to N. Thus $4151 = 4^5+1^5+5^5+1^5$ (N=4151, k=4, n=5). Find more such numbers.
40. Find all the non-negative integers you can get by using two 2's and usual arithmetic symbols (+, —, ×, division, decimal point, square root, reciprocal).
41. If the length of the edge of a cube is increased by

a inches, then the increase in surface area in square inches = the increase in volume in cubic inches. What is the maximum value of a in inches ?

42. Express 1/7, 2/7,....., 6/7
1/17, 2/17,....., 16/17
1/29, 2/29,....., 28/29
as recurring decimals and comment on the patterns you get.

43. The numbers 1, 2, 3, 5, 8, 13, 21, 34, 53, 89, 144, 233, 377, 610, 987, 1597.... — in which each number is the sum of the two preceding numbers are called Fibonacci numbers. Let F_n be the nth Fibonacci number. Verify that
$F^2_{n-1}+F^2_n=F_{2n-1}$, $2F_n-F_{n+1}=F_{n-2}$
$F_1+F_2+.....+F_n=F_{n+2}-1$
$F^3_{n+1}+F^3_{n+2}-F^3_n=F_m$ for some m. Find more such patterns.

44. Show that in a Fibonacci sequence an infinite number of members is divisible by 5 and another infinity of members is divisible by 8.

45. Verify that 1/89 = 0.112358
13
21
34
55
89
144
233
....
= 0.11235955040673
so that in some sense F_{11} generates the Fibonacci sequence.

46. Show that if $N = \ldots\ldots\ldots\ldots a_5a_4a_3a_2a_1$ where a_1's are digits 0 to 9, is divisible by 7, then so is the number

$$N_1 = a_1 + 3a_2 + 2a_3 + 6a_4 + 4a_5 + 5a_6 + a_7 + 3a_8 + 2a_9 + 6a_{10} + 4a_{11} + 5a_{12} \ldots\ldots\ldots$$

The cycle (1, 3, 2, 6, 4, 5) is called a multiplier cycle for divisibility by 7. Find similar multiplier cycles for testing divisibility by 11, 13, 17, 19, 23, 29. By using these cycles, test the divisibility of 215656441 by each of these numbers.

47. Take an equilateral triangle, divide each side into three equal parts and draw an equilateral triangle on the middle part of each side outside the original triangle to get a twelve-sided polygon.
Repeat the process to get a 48-sided polygon.
If the length of sides of original triangle is a, show that the lengths of sides of subsequent polygons are a/3, a/9, a/27,..... and the perimeters of the original triangle and polygons are 3a, 12.a/3, 48.a/9, 192.a/27....., which form a GP with common ratio 4/3. Show that the perimeter approaches infinity as the number of operations increases to infinity. Find also the areas of the polygons and show that the area of polygon approaches a finite limit as the number of operations increases to infinity, so that ultimately we get a polygon with infinite perimeter and finite enclosed area.

48. Take a triangle, a quadrilateral, a pentagon, a hexagon, heptagon (all convex). Make 100 copies of each. Try to cover the ground with each. In which cases do you succeed and in which cases do you fail ?

49. (a) Out of the figures 1, 2, 3, 4 given below, which ones can you draw without lifting the pencil from the paper. Try to find a common property of these figures.

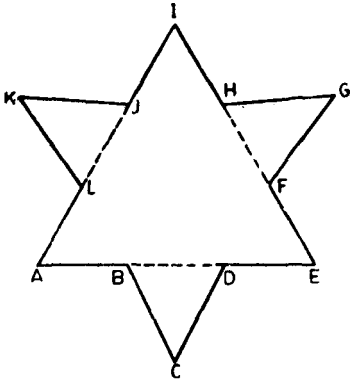

Fig. 1

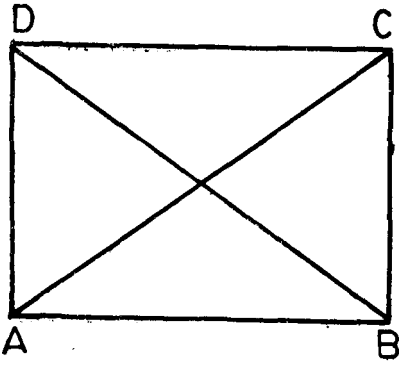

Fig. 2

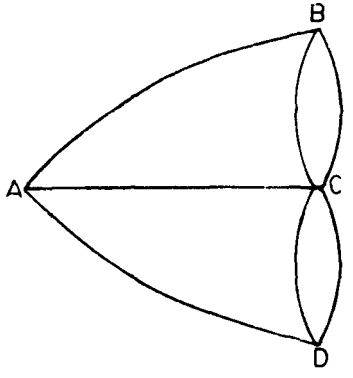

Fig. 3

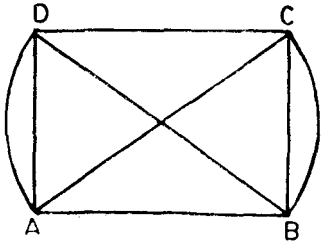

Fig. 4

(b) Can you join A, B, C with each of 1, 2, 3 (Fig. 5) by paths, straight or curved, which do

not intersect,

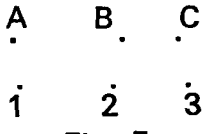

Fig. 5

50. There are many methods of sending messages secretly. One method is to replace letters e.g. we can replace A by C, B by D,....., X by Z, Y by A, Z by B or we can reverse the letters in each word or we can replace letters by certain symbols agreed to in advance. Thus

I LOVE MATHEMATICS can become K NQ X G OCVJGOCVKEU

MATHEMATICS IS FUN can become SCITAMEHTAM SI NUF

Another simple method is to write the message in rectangular form according to columns and send it according to rows. Thus the message

ENEMY PLANS ATTACK AT DAWN is written as

E Y N T A W
N P S A T N
E L A C D Y
M A T K A Z

Here Y, Z are dummy letters. It is coded and sent as follows:

EYNT AWNP SATN ELAC DYMA TKAZ

Coding and decoding are highly interesting mathematical problems. Code the following message according to each of the above methods

PROBLEM SOLVING IS THE HEART OF MATHEMATICS.

SECTION B

SOLUTIONS OF PROBLEMS AND RELATED PROBLEM SETS

Problem 1

What was the last year in which there was no Friday the thirteenth ?

Solution 1

The direct method is to take the calendars of the last twenty or thirty years and find if there is any year which had no Friday, the thirteenth. We may even take a calendar for the whole 20th century and we may find that every year has at least one Friday the thirteenth. This leads us to the conjecture that perhaps there was no year in which there was no Friday the thirteenth, but this does not constitute a proof. We will have to look for calendars of earlier centuries. Constructing these calendars and then scanning them will be a time-consuming process.

We therefore give up the direct method and try to use our greatest asset viz. our power of reasoning systematically and logically.

We first note that if 13th of a month is to be a Friday, the 1st of the month must be a Sunday, so question then becomes : Is there a year in which no month starts with a Sunday ? Now there are two types of years viz. leap years and non-leap years. It can be easily veri-

fied that in a non-leap year, months of January, February, March, May, June, September, and October start on different days of the week and as such there is at least one month which starts on a Sunday.

Again it can be verified that in a leap year, the months of January, February, March, May, June, August and September start on different days of the week and therefore there must be at least one month which starts on a Sunday.

Thus in every year, whether leap year or not, there is at least one month which starts with a Sunday and in which 13th is a Friday.

In fact in many years there may be two Fridays the 13th i.e. there can be two months starting with a Sunday.

It may be noted that in this problem, we were asked to find the last year in which there was no Friday the 13th and we ended up by showing that there was no such year. Similarly in many problems, we may be asked to find a number satisfying certain conditions and we may end up by showing that no such number exists.

Problem Set 1

(i) What was the last year in which there were two Fridays the 13th ?

(ii) Can there be a year with three Fridays the thirteenth ?

(iii) Can there be a year in which there is no Friday the thirty-first ?

(iv) Which day of the week was most frequent in the 20th century ?

(v) Make a calendar for the 21st century for your drawing room.

(vi) How many Januaries in the 21st century will have Friday the thirteenth ?

Problem 2

How many squares (of all sizes) are there on a chess board ?

Solution 2

We can try to count 1×1, 2×2, 3×3, 4×4, 5×5, 6×6, 7×7, and 8×8 squares. Unless we try a systematic procedure, we are bound to miss some squares. We therefore introduce a systematic procedure.

We assign a number to every square ; we assign it the number written on its upper left-hand corner (Fig. 6).

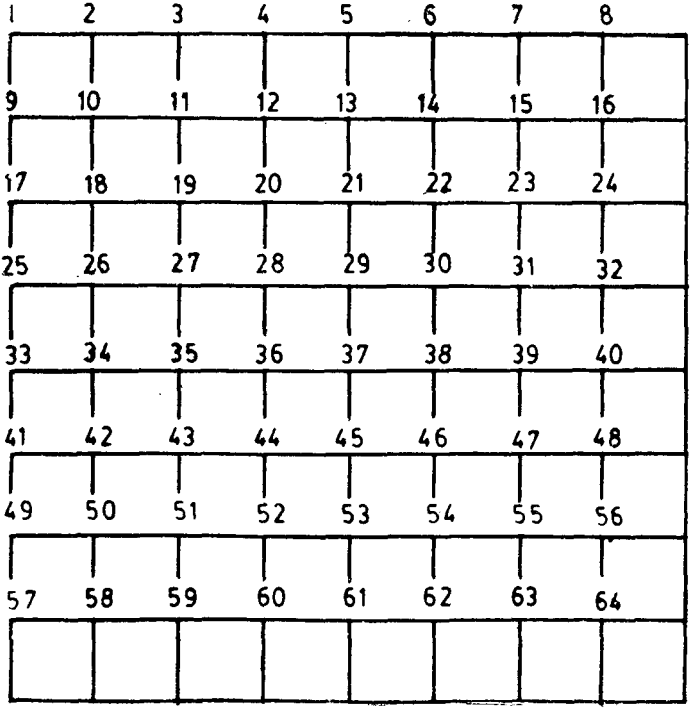

Fig. 6

There are *64* squares of size 1×1 and their upper left hand corners are numbered 1—64.

There are *49* squares of size 2×2 and their upper left-hand corners are numbered 1—7, 9—15, 17—23, 25—31, 33—39 and 41—47.

There are *36* squares of size 3×3 and their upper left-corners are numbered 1—6, 9—14, 17—22, 25—30, 33—38 and 41—46.

There are *25* squares of size 4×4 and their upper left-hand corners are numbered 1—5, 9—13, 17—21, 25—29 and 33—37.

There are *16* squares of size 5×5 and their upper left-hand corners are numbered 1—4, 9—12, 17—20 and 25—28.

There are *9* squares of size 6×6 and their upper left-hand corners are numbered as 1—3, 9—11, and 17—19.

There are *4* squares of size 7×7 and their upper left-hand corners are numbered as 1—2, 9—10.

There is *1* square of size 8×8 and its upper left-hand corner is numbered as 1.

Thus the total number of squares $=$
$8^2+7^2+6^2+5^2+4^2+3^2+2^2+1^2=64+49+36+25+16+9+1=204$.

By using a logical and systematic procedure, we have not only solved the given problem, but we have developed a technique which can solve many similar problems. Mathematics is certainly one of the best tools for learning logical and systematic thinking.

Problem Set 2

(i) How many squares of all sizes are there in 9×9, 10×10,....., 50×50 square boards ?

(ii) How many rectangles of all sizes are there on a chess board ?

(iii) How many rectangles of all sizes are there on a $m \times n$ board where m and n are any positive integers ?

(iv) How many triangles of all sizes are there in figure 7 ?

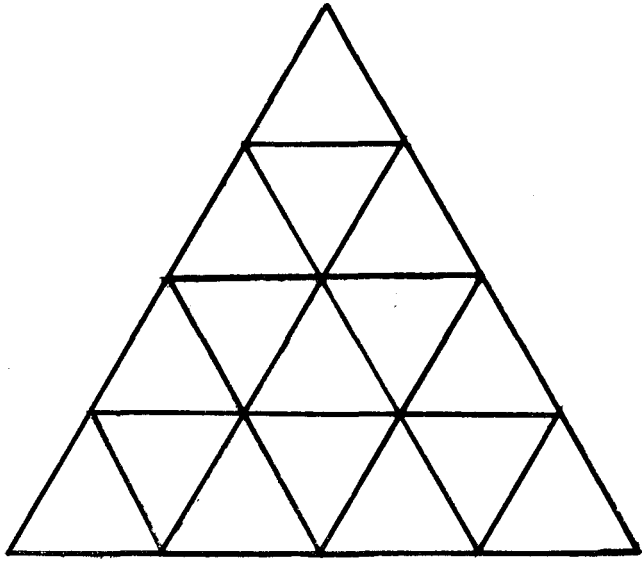

Fig. 7

(v) How many triangles of all sizes will be formed if each side of the original equilateral triangle is divided into 5, 6, 7 or 8 parts ? What pattern do you observe ? What happens if the original triangle is not equilateral ?

Problem 3

What is the maximum number of regions into which a plane is divided by five lines, no two of which are

parallel and no three of which intersect in a common point ?

Solution 3

We draw one line I, it divides the plane region into 2 parts.

We draw a second line II, the plane region is now divided into 4 parts. (Fig. 8).

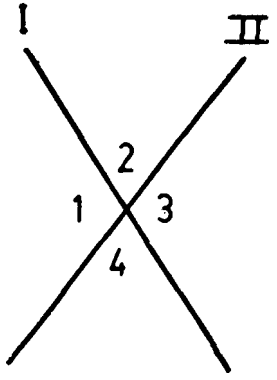

Fig. 8

We draw a third line III, the plane region is now divided into 7 parts. (Fig. 9).

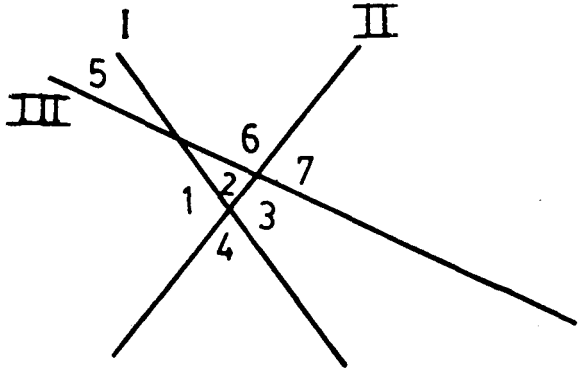

Fig. 9

We draw a fourth line IV, the plane region is now divided into 11 parts. (Fig. 10).

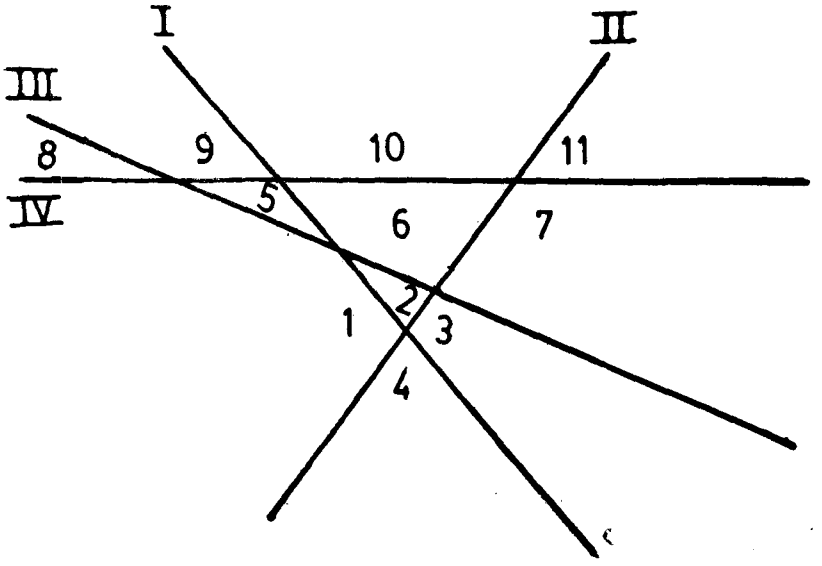

Fig. 10

We get the table :

No. of lines	1	2	3	4 :	5	6	7
No. of regions	2	4	7	11 :	15	22	29

The number of regions increases by 2, 3, 4.....Using this, we can complete the table.

At any stage we find that the number of regions obtained by adding a line is one more than the numbers of lines. Thus if u_n is the number of regions, $u_{n+1} = u_n + n + 1$. Also $u_1 = 2$. We can now complete the table by putting $n = 1, 2, 3, \ldots\ldots\ldots m$ in succession in this formula.

Problem Set 3

(i) What is the maximum number of regions generated if the number of lines is 5, 6, 7, 8, 9, ... ?

(ii) What is the maximum number of regions generated if the number of lines is n, where n is any positive number ?

(iii) What is the number of regions generated if the lines are all parallel, but distinct ?

(iv) What is the number of regions generated if the lines are all concurrent, but distinct ?

(v) What is the number of regions, if we draw n concentric circles ?

(vi) What is the number of regions if there are n non-concentric non-intersecting circles (n = 3, 4, 5, 6.....) ?

(vii) What is the maximum number of regions if there are n non-concentric non-intersecting circles ?

(viii) Generalise the above results to the case when we have planes in three dimensional space (instead of straight lines in two-dimensional space) and the planes are not parallel and no three of them have a common line of intersection.

Problem 4

If in a party everybody shakes hands with everybody else and there are 36 hand-shakes in all, how many persons are there in the party ?

Solution 4

If there are 2 persons A, B there is 1 handshake AB.

If there are 3 persons A, B, C there are 3 handshakes AB, AC, BC.

If there are 4 persons A, B, C, D there are 6 handshakes AB, AC, AD, BC, BD, CD.

If there are 5 persons A, B, C, D, E there are 10 handshakes AB, AC, AD, AE, BC, BD, BE, CD, CE, DE.

and so on. We get the table

No. of persons	2	3	4	5 :	6	7	8	9
No. of handshakes	1	3	6	10 :	15	21	28	36

We find that in the second line we are adding numbers 2, 3, 4, 5, 6,.....respectively.

We find that for 36 handshakes, we require 9 persons in the party. Alternatively if there are n persons, each shakes hands with the other n—1, so that the number of handshakes is n (n—1), but the two handshakes A with B and B with A are the same. As such the total number of handshakes is $\frac{n\,(n-1)}{2}$

Since the total number of handshakes is 36, we get :

$$\frac{n\,(n-1)}{2} = 36 \text{ or } n\,(n-1) = 72 \text{ or } n=9.$$

Problem Set 4

(i) What is the number of persons if the number of handshakes is 1225 ?

(ii) There are 25 men and 25 women at a party. How many distinct dancing pairs are possible ?

(iii) There are 20 persons present at a party. How many distinct bridge-playing teams can be formed ?

Problem 5

In a regular polygon of seventeen sides, each vertex is joined to the other sixteen by a line which may be

red, blue or green. Show that there will be at least one triangle with all its sides of the same colour.

Solution 5

The direct method would be to draw a regular polygon of 17 sides and join each vertex to the other 16, so that the number of lines is 17×16, but since we get the same line whether we join A to B or B to A, the total number of lines is $17 \times 8 = 136$. Each line can be coloured in 3 different ways to give 408 different polygons. We may then see all these polygons and verify the theorem stated.

This procedure will be time-consuming, mechanical and unscientific.

We therefore use logical and systematic reasoning.

Out of sixteen lines drawn from a given vertex, at least six must be of the same colour, say red (why) ; out of the five lines joining any of the six identified vertices to the other five, one may be red in which case we get a red triangle and the result is proved. If none of these is red, at least three must be of the same colour (why), say blue. The three lines joining the three vertices so obtained may all be green in which case a green triangle is obtained and if one of these three lines is blue, a blue triangle is obtained. In every case, at least one triangle with all three sides of the same colour is obtained.

It may be noted that a problem which would have taken many days by the direct method of drawing polygons and lines and still may not give the correct result if the constructions are not accurate, can be solved by systematic reasoning in a matter of minutes. This demonstrates the power of logic and mathematics.

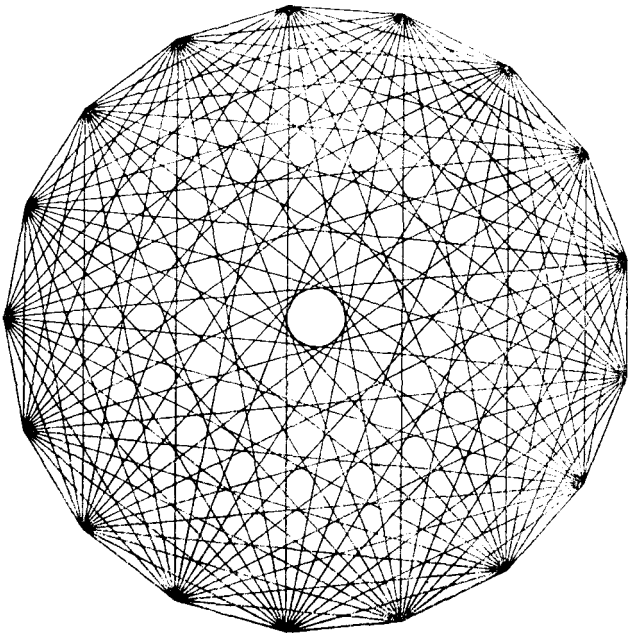

Fig. 11

Problem Set 5

(i) Formulate and solve the corresponding problem when the number of colours is 2, 4 or 5.

(ii) How is the problem affected if the polygon is not regular or if its edges are not straight lines ?

(iii) Show how you would construct a regular polygon of 17 sides with ruler and compasses only ? Gauss was the great mathematician who gave the method for this for the first time at the early age of 17.

(iv) Draw some of the 408 polygons and count for each the number of triangles with all sides of same colour, and all sides with different colours.

Problem 6

Express all numbers from 1 to 50 as the sum of at most four square numbers.

Solution 6

$1=1^2$, $2=1^2+1^2$, $3=1^2+1^2+1^2$, $4=2^2$, $5=2^2+1^2$, $6=2^2+1^2+1^2$, $7=2^2+1^2+1^2+1^2$, $8=2^2+2^2$, $9=3^2$, $10=3^2+1^2$,....,$15=3^2+2^2+1^2+1^2$, .. ,$18=3^2+3^2$,.... ....$20=4^2+2^2$, ... $23=3^2+3^2+2^2+1^2$, ...$27=5^2+1^2+1^2$, $28=3^2+3^2+1^2$, ... $31=5^2+2^2+1^2+1^2$...., $34=5^2+3^2$, ..., $39=6^2+1^2+1^2+1^2$, ..., $43=5^2+4^2+1^2+1^2$, ... $46=6^2+3^2+1^2$, $47=6^2+3^2+1^2+1^2$, $48=6^2+2^2+2^2+2^2$, $49=6^2+3^2+2^2$, $50=5^2+5^2$.

We find that only a few numbers require four squares, though no number requires more than four squares.

The Greeks had made a conjecture that *every number* can be expressed as sum of *at most* four squares. This was proved by Lagrange (1736—1813).

As soon as this problem was solved, other problems arose. What is the minimum number of cube numbers or fouth powers of numbers etc. that we require for expressing *every* number as a sum of. Waring conjectured that every number can be expressed as sum of 9 cubes or 19 biquadratics and so on. This conjecture has been finally proved by the combined efforts of Indian, Chinese, Russian, British, French and American mathematicians. However the final proofs rest on a conjecture which is almost certainly true, but has not been proved rigorously. Moreover the proof for fourth powers requires the use of computers.

Even this is remarkable if we remember that Waring's

conjecture implies an infinity of theorems, each concerning an infinity of numbers.

Problem Set 6

(i) Express every number from 51 to 100 as the sum of at most 4 square numbers.

(ii) Express every number from 100 to 150 as the sum of at most 9 cubes.

(iii) Express every number from 150 to 160 as the sum of at most 19 biquadratics.

(iv) How many square (cube) numbers you can express as the sum of two squares (cube) numbers ?

(v) Express every number as the sum of non-decreasing natural numbers. Thus $1=1$, $2=1+1$, $3=3=2+1=1+1+1$, $4=4=3+1=2+2=2+1+1=1+1+1+1$. We denote by p (n) the number of ways in which n can be so expressed and call these ways as partitions. Show that
p (1)=2, p (2)=1, p (3)=3, p (4)=5, p (5)=7, p (6)=11, p (10)=42.
It can be shown that p (100)=190569292, p(200) = 397299929388. Hardy and Ramanujan gave the general formula for p (n). If it takes 1 second to write one partition of 200, how much time it would take to write all partitions of 200 and then count them. While the brute-force method would take billions of years for finding the number of partitions of a number, Hardy-Ramanujan formula enables us to calculate this number in about a minute only ! What greater tribute can there be to the human mind ?

Problem 7

Find a square number which is double of another square number.

Solution 7

We first try the direct method. We write all square numbers and find if there is a square number which is double of another square number. The square numbers are :

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 381, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961, 1024.....

None of odd square numbers can be double of another square number. For even numbers, 16 is double of 8, but 8 is not a square number; 36 is double of 18, but 18 is not a square number and so on. We do not find any square number in the above list which is double of another square number. This does not prove that there will be no such number, since there is an infinity of square numbers.

The direct method may require us to try an infinity of numbers and this may require an infinite time.

Let us try logic and ingenuity.

We have to solve $x^2 = 2y^2$ in integers. If x and y have a common factor, we can cancel it out, so that we can assume that x and y have no common factor.

Since x^2 has 2 as a factor, x^2 is an even number, so that x is an even number, since the square of only an even number can be even. Let $x = 2z$, then we get $4z^2 = 2y^2$ or $y^2 = 2z^2$, so that y^2 is an even number and therefore y is an even number, but x and y cannot both be even numbers, since these have no factor in common. This

leads to a contradiction. Therefore $x^2=2y^2$ has no solution in integers. As such, there is no square number which is double of another square number.

Problem Set 7

(i) Show that there is no square number which is 3 times another square number.

(ii) Show that there is an infinity of square numbers, each of which is 4 times another square number.

(iii) Show that there is no square number which is 3 or 5 or 6 or 7 or 8 or 10 times another square number.

(iv) Show that there is no square number which is k times another square number, when k is not itself a square number.

(v) Show that there is an infinity of square numbers each of which is k^2 times another square number when k is any natural number.

(vi) Show that there is no another number which is k times another cube number, if k is not a perfect cube number.

(vii) Show that there is an infinity of cube numbers, each of which is k^3 times another cube number, where k is any natural number.

(viii) Show that there is no mth power of a natural number which is k times another mth power of a natural number, unless k is itself an mth power of a natural number.

Problem 8

One solution in integers of the equation $x^2+y^2+z^2=xyz$ is (3, 3, 6). Find ten more solutions in integers of this equation.

Solution 8

Let a, b, c be a solution of the given equation so that

$$a^2 + b^2 + c^2 = abc,$$

then it is easily verified that bc—a, b, c is also a solution, since this requires

$$(bc-a)^2+b^2+c^2 = (bc-a)\,bc$$

or $$a^2+b^2+c^2 = abc,$$

which is true

Thus (a, b, c) is a solution ⇒ (a, b, c), (b, a, c), (a, c, b), (b, c, a), (c, a, b), (c, b, a)

(bc—a, b, c). (ac—b, a, c), (bc—a, c, b), (ca—b, c, a) (ab—c, a, b), (ab—c, b, a) are solutions.

This gives the solutions (3, 3, 6), (3, 6, 3), (6, 3, 3), (15, 3, 6), (15, 6, 3), (6, 15, 3) (6, 3, 15), (3, 15, 6), (3, 6, 15)

From these we can generate more solutions.

Problem Set 8

(i) One solution of $x^2+y^2=z^2+u^2$ is (2. 2, 2, 2). Obtain an infinity of solutions of this equation.

(ii) Extend the above results to equations in five or more unknowns.

Problem 9

Take any three points on the circumference of a circle. Join these by straight lines. Show that you get four regions inside the circle. Similarly take four and five points and show that you can get eight and sixteen regions respectively. Now take six points on the circumference of the circle and join

them by straight lines. What is the largest number of regions that you can get ?

Solution 9

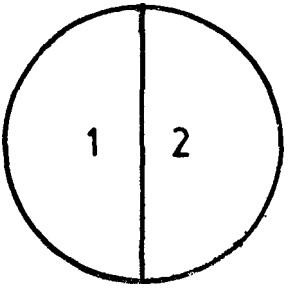

Fig. 12

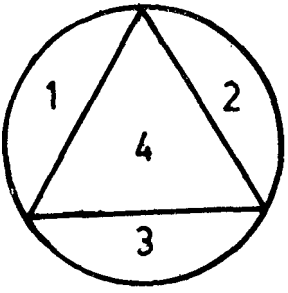

Fig. 13

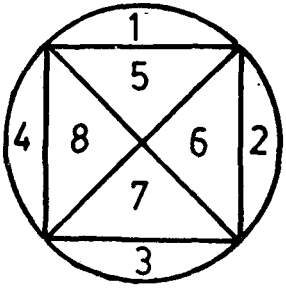

Fig. 14

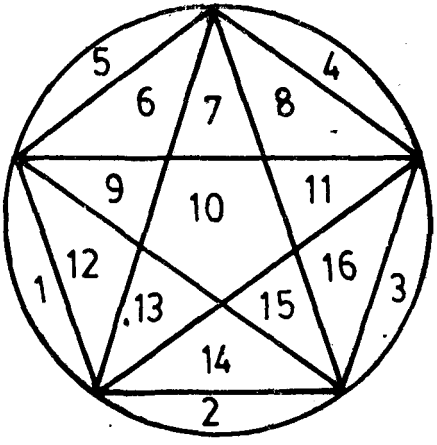

Fig. 15

From the above figures and figure 16 on next page, we get the table

No. of points	2	3	4	5	6
No. of regions	2	4	8	16	31

From the first four values, it may appear that the number of regions for n points is 2^{n-1}, and this formula

suggests that we should get 32 regions in the case when the number of points is 6. In fact this number is 31 and

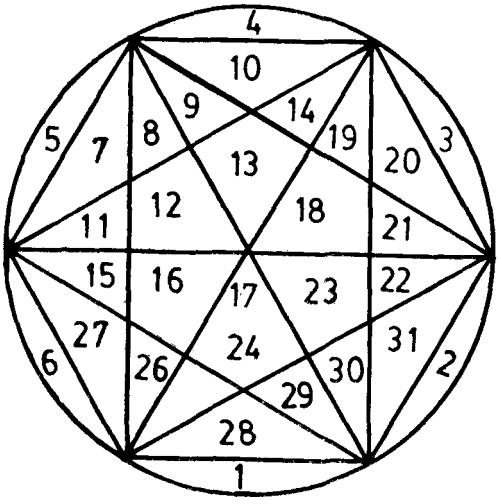

Fig. 16

as such the number of regions is not in G.P. Actually the number of regions is given by

$$\frac{1}{24}(n^4-6n^3+23n^2-18n+24)$$

which gives the correct value 31 tor $n=6$. This formula has been proved rigorously. This gives the correct value 31 for $n=6$ and the correct value 57 for $n=7$ and so on. The proof of the above formula is given in the author's book 'Reflections of a Mathematician', pages 337-347, Arya Book Depot, Delhi.

This result shows that the obvious formula may not be the correct formula, unless it can be established mathematically. (For more examples of this type, see the author's book 'Nature of Mathematical Thinking', published by Frank Bros, Delhi, pages 15-22).

Problem Set 9

(i) Does the formula given above hold if the points are taken on any closed convex curve (say an ellipse) rather than on a circle ?

(ii) In the above figures, show that the number of vertices, edges (straight or curved) and regions (including the one outside the circle) are as follows :

Figure	1	2	3	4	5
No. of vertices V	2	3	5	10	20
No. of edges E	3	6	12	25	50
No. of regions R	3	5	9	17	32

In each case show that $V-E+R = 2$

(iii) If n points are chosen on the circle, find the number of vertices, edges and regions and show that $V-E+R = 2$ holds for all n.

(iv) Consider the following four dimensional solids

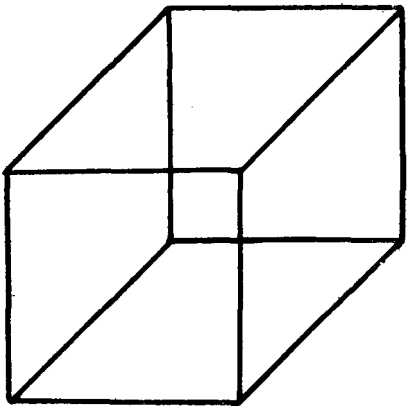

Fig. 17 (cube)

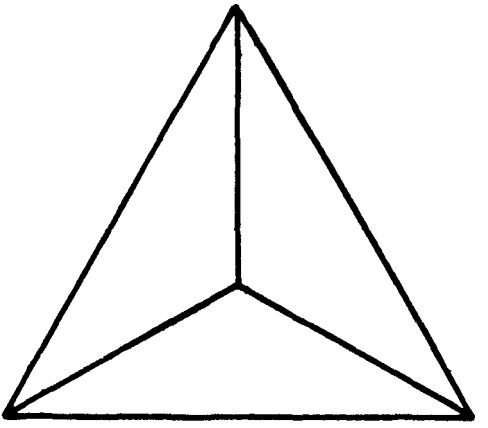

Fig. 18 (tetrahedron)

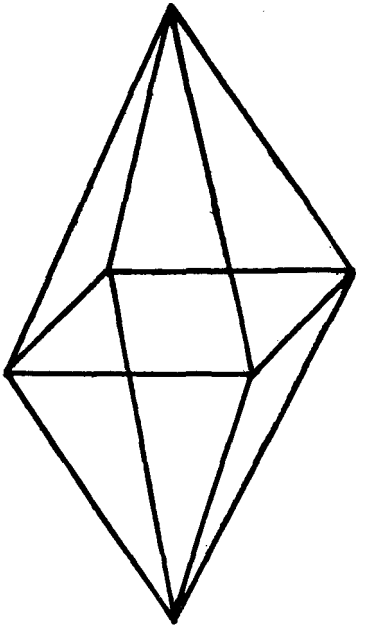

Fig. 19 (octahedron)

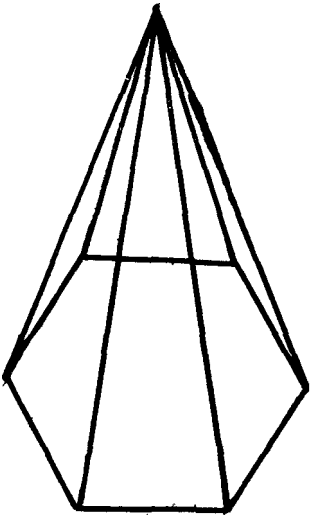

Fig. 20
(hexagonal pyramid)

Find the number of vertices V, edges E, faces F in each case and verify that $V-E+F = 2$.

Problem 10

How many rectangles are there with perimeter 36 inches ? Which of these encloses the largest area and why ?

Solution 10

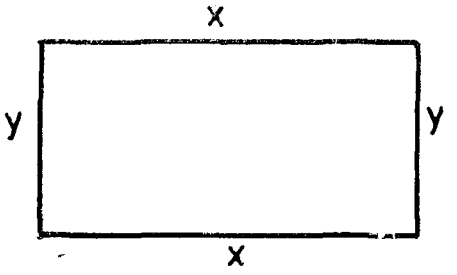

Fig. 21

Let x" be the length and y" be the breadth of the rectangle, than the perimetar $= 2x+2y = 36$,

so that $x+y = 18$

Area of the rectangle $= xy = x(18 - x)$ square inches

$= 81 - (9 - x)^2$ square inches

$\leqslant 81$ square inches

Thus the area is maximum when $x = 9''$, $y = 9''$

i.e. when the rectangle is a square.

The same argument will show that of all reactangles with a given perimeter, the square has the largest area.

Problem Set 10

(i) Show that if the perimeter of the rectangle is P", then the area is maximum when each side is P/4" and as such the maximum area is $P^2/16$ square inches.

(ii) Show that of all reactangles with given area A

square inches, the square has the minimum perimeter and this minimum perimeter is $4\sqrt{A}$ inches.

(iii) Assuming that the arithmetic mean $(x_1+x_2+....+x_n)/n$ of n positive numbers $x_1, x_2.....,\ x_n$ is always $\geqslant$ the geometric mean $\sqrt[n]{x_1, x_2, ...x_n}$ of these numbers and the equality sign holds if and only if $x_1 = x_2 = ... = x_n$, show that of all rectangular parallelopipeds with given perimeter, the cube has the maximum volume and of all rectangular parallelopipeds with given volume, the cube has the minimum perimeter.

Problem 11

The digits of a certain integer are rearranged. Of all the numbers so obtained, the largest is divided by the smallest. Show that if the ratio is an integer, then it must be a square number.

Solution 11

If possible, let the ratio be k (k = 2, 3, 4, 5, 6, 7, 8, 9). then

$a_o..... a_n = k\ (a_n.....a_o)$; $a_o, ..., a_n$ are all digits, so that

$$10^n a_o + 10^{n-1}a_1 + ... + a_n = k\ (10^n a_n + 10^{n-1}a_{n-1} ..... + a_o)$$

This gives $a_n = ka_o$ or $a_n + 10k' = ka_o$ (k' is a positive integer)

and $a_o = ka_n$ or $a_o = ka_n + 1$

Four possibilities are : (i) $a_n = ka_o,\ a_o = ka_n$

(ii) $a_n = ka_o,\ a_o = ka_n + 1$

(iii) $a_n + 10k' = ka_o,\ a_o = ka_n$

(iv) $a_n + 10k' = ka_o,\ a_o = ka_n + 1$

The two equations of (i) are in consistent
The two equations of (ii) are also inconsistent
(iii) gives $(k^2 - 1)a_n = 10\,k'$, $a_o = ka_n$

$k = 2$ gives $3\,a_n = 10\,k'$, This is not possible

$k = 3$ gives $8\,a_n = 10\,k'$ which gives $k' = 4$, $a_n = 5$, $a_o = 15$ which is not possible because a_o is an integer ≤ 9

$k = 4$ gives $15\,a_n = 10\,k'$ gives $k' = 3$, $a_n = 2$, $a_g = 8$

$k = 5$ gives $24\,a_n = 10\,k'$ This gives no solution

$k = 6$ gives $35\,a_n = 10\,k'$ or $7\,a_n = 2\,k'$ gives $k' = 7$, $a_n = 2$, $a_o = 12$ which is not possible as $12 > 9$

$k = 7$ gives $48\,a_n = 10\,k'$ which is not possible

$k = 8$ gives $63\,a_n = 10\,k'$ which is also not possible

$k = 9$ gives $80\,a_n = 10\,k'$ gives $k' = 8$, $a_n = 1$, $a_g = 9$

$\therefore$ $k = 4$ and $k = 9$ give the two possible solutions
(iv) gives $(k^2 - 1)\,a_n = 10\,k' - k$, $a_o = ka_n + 1$

$k = 2$ gives $3\,a_n = 10\,k' - 2$, gives $k' = 2$, $a_p = 6$, $a_o = 12$, not possible.

Similarly it can be shown that $k = 3, 4, 5, 6, 7, 8, 9$ give no possible solutions.

As such the only possible values of the ratio k are 4 and 9 which are both square numbers. The two solutions are :

$k = 4$, $a_o = 8$, $a_n = 2$

$k = 8$, $a_o = 9$, $a_n = 1$

and set of solutions is given by $8712/2178 = 4$, $9801/1089 = 9$. We have also solutions $87128712/21782178 = 4$, $98019801/1089089 = 9$, $871287128712/217821782178 = 4$, $980198019801/108910891089 = 9$. We have an infinity of such solutions.

Problem Set 11

(i) If $a = 1089 = (33)^2$, show that the numbers are $8a/2a$, $9a/a$.

(ii) Can you find numbers other than those given above which have the stated properties ?

(iii) Show that no number when divided by the number obtained by reversing its digits can give the quotient as a perfect cube.

Problem 12

A natural number (other than 1) which is divisible only by itself or unity is called a prime number. Thus 2, 3, 5, 7, 11, 13, 17..... are prime numbers. How many prime numbers are there in all ? How many prime numbers are less than 100 ? How many are less than 1000 ? Give ten consecutive numbers, none of which is a prime number.

Solution 12

Every number is either prime i e. it has 1 and itself as its only factors or it is composite i.e. it has at least two factors less than itself. One of the factors of a composite number must be less than its square root (why ?).

To find all prime numbers less than 1000, we write all numbers from 1 to 1001. Since $\sqrt{1000} = 31.6227766$, we find all prime numbers less than this square root. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31. From the list of all numbers from 1 to 1000, we strike off all multiples of these eleven prime numbers and we are left with the following 168 prime numbers.

LIST OF PRIME NUMBERS LESS THAN 1000

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233, 239, 241, 251, 257, 263, 269, 271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349, 353, 359, 367, 373, 379, 383, 389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461, 463, 467, 479, 487, 491. 499, 503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593, 599, 601, 607, 613, 617, 619, 631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709, 719, 727, 733, 739, 743, 751, 757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839, 853, 857, 859, 863, 877, 881, 883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977, 983, 991, 997,

To find whether a given number N is prime, we test whether it is divisible by all primes less than the square root of N. It will be prime if it is not divisible by any one of these.

This process of striking off multiples of primes is called sieve of Erosthenes since it was devised by Erosthenes more than 2000 years ago. We now have quicker methods and these have been used to find a million primes and even more. In fact we have the table

N	100	500	1000	10000	50000	100000	500000
Number of primes $\leq$N	25	95	168	1229	5133	9592	41538

N	1,000,000	10,000,000,
Number of primes $\leq$ N	78498	664,579

When we take N=100, 25% of numbers are prime, but when N is 10 million, less than. 07% are prime. Thus

primes become scarcer and scarcer as we go ahead. Will there be a stage when there will be no primes ? Euclid proved more than 2000 years ago, in a very simple manner, that there will never be such a stage and however large N may be, there would be primes larger than N; in fact there would be an infinity of primes larger than any given number N, however large N may be.

Euclid argued that if the number of primes is not infinite, it must be finite and there must be a largest prime number say P. If so, we can form a number $(2.3.5.7.11.....P)_+ 1$ i.e. product of all finite number of primes increased by unity. This number is not divisible by any prime number and so it must be prime. This gives us a larger prime than P which contradicts the asumption that P is the largest prime. Thus there is no largest prime number.

However at any time there is a *largest known prime number.* Today it is $2^{216091}-1$ which is a 65050 digit number. With powerful computers we may find still larger prime numbers, but we will never be able to find the largest prime number, since such a number does not exist.

It is a remarkable tribute to the human genius that today we can list more than a million primes (Bytes vol 10) and we can find prime numbes of more than 65000 digits.

Even more remarkable is the fact that we have got a vere good approximation formulae for the number of primes less than N given by

$$\frac{N}{1+1/2+1/3+.....i/N}$$

It is not exact, but approximations become better and

better as N increases and the error is less than .05% when N is 10 million.

The primes do not occur regularly. Thus in the decades 201-210, 321-330, 511-520, 531-540, 621-630, 841-850, 891-900, 1071-1080, 1131-1140, 1261-1270, 1331-1340, 1411-1420, 1461-1470, there are no primes. at all, while many decades contain as many as 4 primes, Such rare prime decades include : 2 3 5 7, 11 13 17 19, 101 103 107 109 191 193 197 199, 821 823 827 829, 1481 1483 1487 1489, 1871 1873 1877 1879, 2081 2083 2087 2089, 3251 3253 3257 3259, 3461 3463 3467 3468. Of course no decade can contain more than 4 primes (why ?)

Problem Set 12

1. Show that the decade 1461-1470 contains no prime
2. Show that the decade 3461-3470 contains 4 primes
3. Show that no decade can contain five or more primes
4. Show that $2^{216091}-1$ has 65050 digits
5. Find $N/(1+1/2+1/3+.....1/N)$ for $N=25$, 50
6. Verify for the first ten prime numbers Wilson's theorem which states that when $1\times2\times3+.....\times(p-2)$ is divided by p, where p is a prime number, the remainder is always unity
7. How many digits will $1\times2\times3\times.....\times(p-2)$ have when p is the largest known prime number. How much time will it take to verify Wilson's theorem for the largest prime number with a computer which can perform a million divisions per second? Is it not remarkable that a theorem which cannot be verified for all p in billions of billions years can be proved mathematically in a very short time?

Note: The number of digits required to express a number N is the integral part of $\log_{10} N + 1$. Also $\log_{10} 2^{216091} = 216091 \times \log_{10} 2 = 2160910 \times 301029995 = 650498.7275$. You can do such problems easily if you have a calculator.

Problem 13

A pair of prime numbers is said to be a twin prime pair if the two numbers differ by only 2. Thus (3, 5), (5, 7), (11, 13), are all twin prime pairs. How many twin prime pairs are there ? Verify that the digit sum of the product of two twin primes is always 8, except for the first pair viz. (3, 5). Also prove this result.

Solution 13

From the table given in solution 12, we find the following twin prime pairs. (3,5), (5,7), (11,13), (17,19), (29,31), (4143), (59,61), (71,73), (101,103), (107,109), (149,151), (179,181), (191,193), (197,199), (227,229), (239,241), (269,271), (281,283), (311,313), (347,349), (419,421), (431,433), (461,463), (521,523), (569,571), (599,601), (617,619), (641,643), (659,661), (809,811), (831,833), (827,829), (857,859), (881,883).

There are no twin primes in the two centuries 701-800, 901-1000. We have the following table for numbers of twin prime pairs and number of primes upto N.

N	100	500	1000	10,000	50,000	100,000
Number of twin primes	8	24	35	205	705	1224
Number of primes	25	85	168	1229	5133	9592

Thus the number of twin prime pairs is much less than the number of primes. Is their number finite or infinite ? This simple question has not yet been answered either in the affirmative or in the negative : Our inability to answer such simple questions makes us feel humble indeed !

To verify the given result, consider $809 \times 811 = 656099$

The sum of its digits is 35. The sum of digits of 35 is 8 and this is divisible by 8. You may verify it for all the twin prime pairs given above [except (3, 5)].

Now all the twin primes are of the form $6n-1, 6n+1$ You can verify this in the above table. To prove this result in general, we note that every number is of the form $6n-2$ or $6n-1$ or $6n$ or $6n+1$ or $6n+2$ or $6n+3$, of which only $6n-1$, $6n+1$ can be twin primes. While every twin prime pair is of the form $6n-1$, $6n+1$, every pair of this form does not give a twin prime pair. Thus for $n=6$ we get 35, 37 which is not a twin prime pair. In fact if this result was true, the number of twin prime pairs would have been infinite and mathematicians would have been saved 2000 years of frustrated effort in not being able to prove whether the number of twin prime pairs is finite or infinite.

Now $(6n-1)(6n+1) = 36n^2 - 1 = 36n^2 - 9 + 8 =$ a multiple of $9 + 8$

Now value of any number $a_k, a_{k-1}\ldots\ldots\ldots\ldots$

$$a_1 a_0 = 10^k a_k + 10^{k-1} a_{k-1} + \ldots\ldots a_0$$

(where $a_0, a_1 \ldots\ldots a_k$ are all digits)

and sum of its digits is $a_k + a_{k-1}\ldots\ldots + a_0$, so that $(10^k a_k + 10^{k-1} a_{k-1} + \ldots\ldots\ldots a_0) - (a_k + a_{k-1} + \ldots\ldots + a_0) = (10^k - 1) a_k + (10^{k-1} - 1) a_{k-1} + \ldots + (10-1) a_1$ which is divisible by 9.

Therefore any number =a multiple of 9 + sum of its digits.

This shows that in the product of (6 n—1) and (6 n+1), the ultimate sum of digits would always be 8.

We heve proved a result so simply for all twin prime pairs, whose number for all we know, may be infinite and which we can not even enumerate.

Problem Set 13

1. Show that the sum of the digits of (3 n—1) (3 n+1) is also ultimately 8.
2. Show that the number of digits of (3 kn—1) (3 kn+1) where k=1, 2, 3, 4..... is always ultimately 8.
3. Show that the sum of digits of (3 kn—2) (3 kn+2) is always ultimately 5.
4. State some more results of the above type.
5. Show that (3, 5, 7) is a prime number triplet. Can there be any other prime number triplet ?

Problem 14

Show that 271 is the first prime factor of 13549729. Which is the next prime factor ?

Solution 14

It is easily verified that 13549729 = 271x49999

There are 57 primes less than 271 and we can show that 13549729 is not divisible by any of these. We can apply tests of divisibility, but it is much easier to use a calculator, though this would also require considerable time.

Now $\sqrt{49999} = 223.6045617$ and this is < 271

Therefore 49,999 is itself the next greater prime factor of 13549729.

Problem Set 14

I. Show that if p is the first prime factor of pxk and k is less than p^2 then k must be a prime number.

II. Given that 420041599 has 31 as a prime factor, find the other prime factors.

Problem 15

If we rearrange digits, 123 can become 132, 213, 231, 312, or 321, we say there are six permutations of digits 1, 2, 3. Write all permutations of digits 1, 2, 3, 4, 5. If all the permutations of 1, 2, 3, 4, 5, 6 are written in increasing order, find the 42nd of these.

Solution 15

There are $1 \times 2 \times 3 = 6$ permutations of the digits 1,2,3 viz, 123, 132, 213, 231, 312, 321 : There are $1 \times 2 \times 3 \times 4 = 24$ permutations of the digits 1,2,3,4 viz. 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 2314, 2341, 2413, 2431, 3124, 3142, 3214, 3241, 3412 3421, 4123, 4132, 4213, 4231, 4312, 4321. There are $1 \times 2 \times 3 \times 4 \times 5 = 120$ permutations of the digits 1, 2, 3, 4, 5.

There are $1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ permutations of the digits 1, 2, 3, 4, 5, 6.

We have to arrange all these 720 numbers in increasing order.

Out of these 720 permutations, 120 start with 1 as the leftmost digit and all these are smaller than those permutations which start with 2,3,4,5,6, as leftmost digits (why ?). Out of the 120 which start with 1, those 24 which start with 12 as the two leftmost digits are the smallest. The next larger are those 24 which start with 13 as the two left most digits and of these 6, which

start with 132 are the smallest, the next six larger are those which start with 134, the next six larger are those which start with 135. These will dispose of 42 permutations. The 42nd, permutation will be the largest of those permutations which start with 135 and this is 135642.

Problem Set 15

i. Write all the permutations of digits 1,2,3,4 in decreasing order.

ii. Write all permutation of letters A,B,C,D.

iii. When all the permutations 1,2,3,4,5.6, are arranged in ascending order, find the 421st term.

iv. Find the total number of permutations of all the 26 letters of the alphabet. If it takes one second to write one permutation, show that it will take you more than 10^{14} years to write all the permutations (assuming you make no mistake).

v. If there are ten men and ten jobs to be done by them, show that the first man can be given any of the 10 jobs and corresponding to each such assignment, the second man can be given any of the remaining 9 jobs so that the total number of ways of assignment of two jobs is 10×9. Show that the total number of ways of assisgnments of all the 10 jobs is $10 \times 9 \times 8 \times 7 \times \ldots\ldots\ldots \times 1 = 3628800 = 3.6 \times 10^6$. Show that the number of seconds in a non-leap year is $60 \times 60 \times 24 \times 365 = 31536000 = 3.1536 \times 10^7$ so that it takes approximately 1/90 of a year to write all assignments. How many years will be required if there are 20 men and 20 jobs First make a guess and then calculate.

Problem 16

Find the next number of the sequence

11, 31, 71, 91, 32, 92, 13, 73, 14, 34, 74, 35, 95, 16, 76, 17, 37, 97, 38, 98

Solution 16

If we reverse the digits every number in the given sequence, we get 11, 13, 17, 19, 31, 37, 41, 43, 53, 59, 61, 67, 71, 73, 79, 83, 89

which are all consecutive prime numbers. The next prime number is 97. As such the next number of the original sequence is 79.

Problem Set 16

i. Show that the number ot reversed primes is infinite.

ii. Give the largest known reversed prime. Will it be obtained by reversing the digits of the largest known prime ?

iii. Give ten numbers which are both primes and reversed primes.

iv. Give some twin reversed prime pairs i.e. pairs of which each member is a reversed prime and the two numbers differ by 2.

v. Show that the difference between a prime number and its reversed number is always divisible by 9.

Problem 17

In how many ways can a square be divided into two congruent parts i.e. such that if one part is put over the other, these will fit completely ?

Solution 17

The four obvious ways are obtained by drawing

lines joining the opposite vertices and lines joining the middle points of the opposite sides.

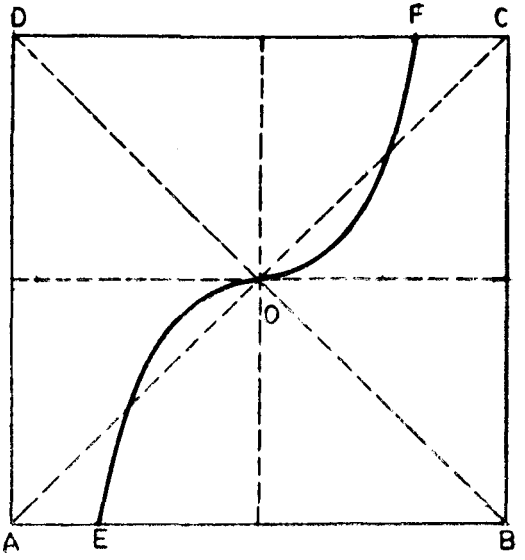

Fig. 22

However the same purpose is achieved by drawing a curve joining E to F where AE = CF and which passes through the centre O of the square ABCD and the curve OF is the reflection of Curve OE in the centre O.

Since we have an infinite number of such curves, we have an infinite number of ways of dividing a square into two congruent parts.

Problem Set 17

i. In how many different ways can you divide a circle into two congruent parts ?

ii. In how many different ways can you divide a cube into two congruent parts ?

iii. In how many different ways can you divide an equilateral triangle into two congruent parts ?

iv. In how many different ways can you divide a tetrahedron into two congruent parts ?

v. In how many different ways can you divide a square or a cube into four congruent parts ?

Problem 18

Which capital letters of the English alphabet can be written in terms of straight line segments only and which numbers can be written in terms of these letters ? In which of these numbers is the number of straight line segments equal to the number itself ?

Solution 18

These letters are A E F H I K L M N T V W X Y Z. The numbers which can be written in terms of these letters are FIVE, NINE, TEN, ELEVEN, TWELVE, FIFTEEN, NINETEEN, TWENTY, TWENTY-FIVE, TWENTY-NINE,.....

Count the number of segments in each of them. It will be found that TWENTY-NINE has exactly 29 segments.

Problem Set 18

i) In digital clocks and calculators, all letters are written in terms of line segments. Thus we have Figure 23.

Fig. 23

Write the number of segments required by each of thetwenty-six letters and arrange letters in increasing order of the number of segments required.

ii) Is there any other number other than twenty-nine which requires the same number of line-segments as the number itself ?

iii) The relative frequencies (probabilities) and the number of line segments required for writing letters of the English alphabet are given in the following table.

Letters	A	B	C	D	E	F	G
Probabilities.	0781	.0128	.0293	.0411	.1305	.0288	.0139
Line Segments	3	6	3	4	4	3	5

H	I	J	K	L	M	N	O
.0585	.0677	.0023	.0042	.0360	.0262	.0728	.0821
3	3	3	3	2	4	3	4

P	Q	R	S	T	U	V	W
.2015	.0014	.0664	.0646	.0902	.0277	.0100	.0149
4	5	5	5	2	3	2	4

X	Y	Z
.0030	.0151	0009
2	3	3

Find the average number of line segments required to write a letter of the English alphabet. Find the corresponding average number for the Hindi alphabet.

Problem 19

Find the smallest number which when divided by some number m from 1 to 20 always leaves m—1 as remainder.

Solution 19

The L C M of 2, 3, 4, 5,.....20 is 232792560
The required number is therefore 232792559

Problem Set 19

i) Find the smallest number which when divided by any number m from 3, 4, 5,.....20
leaves m—2 as remainder.

ii) Find the smallest number which when divided by 2, 3, 4,.....20
always leaves 1 as remainder.

Problem 20

Find a sequence of twelve numbers in which no number is divisible by any number and in which the first and last numbers are as small as possible.

Solution 20

Such a sequence is given by
4, 6, 9, 10, 11, 13, 14, 15, 17, 19, 21, 23

Problem Set 20

i) Find a sequence of 20 numbers starting with 2 satisfying the other conditions ot the problem.

ii) Find a sequence of 20 numbers starting with 5 in which no number divides any other.

Problem 21

A runner notes that wherever he is on the track, his distances from the two flags in the field one km apart, are in the ratio 2 : 1. How long is the track ?

Solution 21

Let A (O, O) and B (a, O) be the positions of the two flags and let P(x, y) be any point on the track, then

PA/PB = k, we get

$$x^2+y^2 = k^2\,[(x-a)^2 + y^2]$$

$$\text{or } (x^2 + y^2)(k^2-1) - 2axk^2 + a^2k^2 = 0$$

$$\text{or } \left(x - \frac{ak^2}{k^2-1}\right)^2 + y^2 = \frac{a^2k^4}{(k^2-1)^2} - \frac{a^3k^2}{(k^2-1)} = \frac{a^2k^2}{(k^2-1)^2}$$

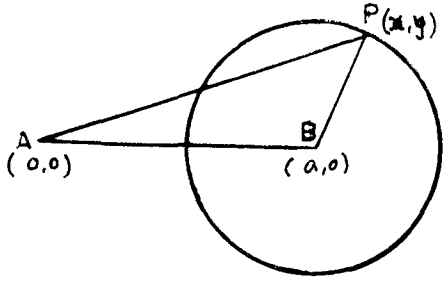

Fig. 24

The track is a circle of radius $\frac{ak}{k^2-i}$

In our case a = 1 km, k = 2, so that

The length of the track $= \frac{2\pi ak}{k^2-1} = \frac{2\pi(1)(2)}{3}$

$= \frac{4\pi}{3}$ kms $= 4.19$ kms

Problem Set 21

i) What is the length of the track if the runner notes that the sum of the distances is always 2 kms ?

ii) What is the length of the track if he notes that the angle subtended by the line joining the flags at his position is always 90° ?

Problem 22

At what temperature will the number of degrees Fahrenheit be the same as the number of degrees Celsius ?

Solution 22

Let F and G denote the the temperatures in the Fahrnheit and Celsius scales, then the relation between F and C is given by

$$\frac{F-32}{180} = \frac{C}{100}$$

In our case, we are given that F and C are equal, so that

$$\frac{C-32}{180} = \frac{C}{100}$$

or $C = -40$,

As such the required temperature is $-40°C$ or $-40°$ F

Problem Set 22

i) At what temperature will the number of degrees Fahrenheit be double the number of degrees Celcius ?

ii) What is $98.4°$ F in the Celcius scale ?

Problem 23

Which is larger in area : a square of 9" side or a circle of diameter 10" ?

Solution 23

The area of a circle of diameter $10'' = \pi (5)^2 = 25\pi$
$= 78.5398163$ sq. inches

The area of a square of side $9'' = 81$ sq. inches.

Therefore the area of the square is larger.

Problem Set 23

i) Will the square be still larger in area if its side was 8.9" long ?

ii) Will the square be still larger in area if its side was 8.85″ long ?

iii) Let the side of the square be 8.8648″ so that its area is 78. 54922383 square inches. The area of the circle of diameter 10″ = 25 π square inches.
= 78.57142853 square inches if $\pi = 22/7$
= 78.53981634 square inches if $\pi = 3.141592654$

Thus it apears that whether circle or square is larger in area depends on the value of π we take.
Comment on this result.

iv) Which will be larger in volume : a cube of side 9″ or a sphere of diameter 10″ ?

Problem 24

Find all the primes less than 1000 which are palindromes i.e. which read the same forwards and backwards ?

Solution 24

From the list of prime numbers in solution 12, we find that palindrome prime numbers less than 1000 are 11, 101, 131, 151, 181, 191, 303, 353, 373, 383, 727, 757, 787, 797, 919, 929

Problem Set 24

i) Show that the left-most digit of a palindrome prime has to be 1 or 3 or 7 or 9.

ii) Is the number of palindrome primes finite or infinite ?

iii) Can there be a palindrome prime pair ?

iv) Show that the set of palindrome prime numbers is the same as the set of those prime numbers which are equal to their corresponding reversed prime numbers.

v) "ABLE WAS I ERE I SAW ELBA" is a palindrome sentence i.e. it reads the same forward or backwards. Write 10 sentences of this type.

Problem 25

For how many values of n is the number $2^{11}+2^8+2^n$ a perfect square ?

Solution 25

$$2^n+2^8+2^n = 2^8(2^3+1)+2^n = 9.2^8+2^n$$
$$= 2^8(9+2^{n-8}).$$

Since 2^8 is a perfect square, $2^{n-8}+9$ should also be a perfect square, so that $2^{n-8}+9 = m^2$ or $2^{n-8} = m^2-9$

or $2^{n-8} = (m-3)(m+3)$.

so that m—3 and m+3 should both be powers of 2. This will be satisfied if m = 5, n= 12.

In fact if $m-3 = 2^k$, $m+3 = 2^{k'}$ then $2^{k'}-2^k = 6$

or $2^k(2^{k'-k}-1) = 6$ or $k = 1, k' = 2$ or

$k = 1, k' = 3, m = 5$, so that

n = 12 is the only value for which $2^8+2^{11}+2^n$ is a perfect square.

Problem Set 25

i) For what value of n is $2^k+2^{k-3}+2^n$ a perfect square ?

ii) For what value of n is $2^{12}+3.2^9+2^n$ a perfect square ?

Problem 26

Find a Pythagorean triplet ($a^2=b^2+c^2$; a, b, c integers), in which all the three a, b, c are odd numbers.

Solution 26

If we search among all the Pythagorean triplets, we do not find a single triplet in which all a, b, c, are odd integers. This does not prove that there will be no such triplet.

We easily see that if b and c are odd then b^2 and c^2 are also odd and $b^2 + c^2$ is even, so that a^2 is even and a is even. As such there can be no Pythogorean triplet in which a,b,c are all odd. We can however have Pythogorean triplets in which a,b, c are all even e.g. $a=10$, $b=6$, $c=8$, but if we cancel out the common factor 2, we get $a=5$, $b=3$, $c=4$ in which two are odd and one is even.

Problem Set 26

(i) Find a Pythagorean triplet a, b, c in which two are even and one is odd.

(ii) Find a solution in integers of

$$a^2=b^2+c^2+d^2$$

Can these be all odd? Give a geometrical interpretation.

(iii) Can you find odd number a, b, c saisfying $a^n+b^n=c^n$ for any integer n ?

Problem 27

The smallest number which can be expressed as the sum of two cubes in two different ways is the Hardy-Ramanujan number $1729=10^3+9^3=12^3+1^3$. Find the next four larger numbers with the same property.

Solution 27

The famous mathematician G.H. Hardy went to see the mathematical genius Srinivasan Ramanujan in a

nursing home in a taxi with number 1729 and remarked that the taxi number was rather uninteresting. Ramanujan at once remarked that the number was very interesting since it was the smallest number which could be expressed as the sum of two cubes in two different ways since

$$1729 = 10^3 + 9^3 = 12^3 + 1^3$$

The question naturally arises whether there are more such numbers. You can verify that

$$4104 = 16^3 + 2^3 = 15^3 + 9^3$$
$$13832 = 24^3 + 2^3 = 20^3 + 8^3$$
$$40034 = 34^3 + 9^3 = 33^3 + 16^3$$
$$64232 = 39^3 + 17^3 = 36^3 + 26^3$$
$$110808 = 48^3 + 6^3 = 45^3 + 27^3$$
$$842751 = 94^3 + 23^3 = 84^3 + 63^3$$

In fact this L.H. number concerns the solution in integers of the equation

$$x^3 + y^3 = z^3 + u^3$$

Since Ramanujan had solved this equation, he knew the smallest number at once.

Hardy asked Ramnujan whether he knew the smallest number which can be experessed as the sum of two fourth powers in two different ways. Ramanujan said that it must be a very large number. In fact Euler had found this number and it was

$635, 318, 687 = 133^4 + 134 = 59^4 + 158^4$

Hardy could not have travelled in a taxi with this number ! Again Euler had tried to solve the following equation in integers

$$x^4 + y^4 = z^4 + u^4$$

Mathematicians do not solve problems by trial and error, but by systematic logical reasoning. However they

can be inspired by such drab things as taxi numbers, car licence numbers or railway carriage numbers.

Once a mathematician preferred to travel in second class than to travel in first class, because the second class carriage number had fascinating properties. When they see a number, they ask themselves: Is it prime ? What are its prime factors? Is it a Fibonacci number or is it a Lucas number? What interesting properties it has ? They have even a theorem that every number is interesting because if not, then let m be the first number which is not interesting; then it is interesting since it is the first number which has no interesting properties !

Problem Set 27

(i) Verify the result given in the above solution.

(ii) What is the smallest number which can be expressed as the sum of two square numbers in two different ways.

(iii) What is the smallest number which can be expressed as the sum of two squares in three different ways ?

(iv) Show that $1729 = 1 \times 7 \times 13 \times 19$ which are four prime numbers in arithmetic progression.

(v) Show that the sum of digits of 1729 is a factor of 1729.

(vi) Show that $1729 = 10^3 + 25^2 + 10^2 + 2^4$

(vii) Show that from the digits of 1729 you can get two reversible prime pairs.

(viii) Write the cubes of all numbers upto 50. Add these in pairs to get 2500 pair sums. Arrange these in order and you will get all the solutions given above except the last. Do you get any more ? This will take you some considerable time if you do this

mentally. It will take you much less time if you do it with a calculator. If you use a computer, it will do the much larger problem of cubing all numbers upto 10^6, adding these in pairs, arranging these in order and noting repetitions of numbers, in a matter of seconds. This looks a miracle. However no computer can give you all solutions in integers of $x^3+y^3=z^3+u^3$, but a mathematician can !

Problem 28

28. In the space-age magic square

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

the sum of the elements of every row, column, diagonal is 34. The sum of the four corner elements and the sum of the four central elements is also 34. The sum of the four elements in each of the four blocks into which it is divided by the two central lines is also 34. There are in all 86 ways in which the sum of four elements situated at the vertices of a regular figure (square, rectangle, parellelogram, y-figure etc.) is 34. Find at least forty of these. If you can find all the 86. write to the publisher who will send a free copy of the Mathematical Enjoyment II to the first five persons who independently send all the 86 solutions.

Solution 28

$34=16+3+2+13 \quad =5+10+11+8$

$=9+6+7+12 \quad =4+15+14+1$

$=16+5+9+4 \quad =3+10+6+15$

$=2+11+7+14$ $=13+8+12+1$
$=16+10+7+1$ $=13+11+4+6$
$=1+4+16+13$ $=2+13+12+7$
$=16+13+1+4$ $=10+11+7+6$
$=2+7+9+16$ $=3+12+14+5$
$=16+3+10+5$ $=2+13+8+11$
$=9+6+15+4$ $=7+12+1+14$
$=3+5+14+12$ $=2+8+15+9$
$=16+2+7+9$ $=3+13+12+6$
$=5+10+15+4$ $=11+8+1+14$
$=3+8+14+9$ $=1+11+16+6$
$=9+10+8+7$ $=5+11+12+6$
$=4+7+13+10$
$=3+2+14+15$ $=5+8+12+9$
$=3+6+9+16$
$=15+12+2+5$ $=11+14+4+5$
$=10+8+1+15$
$=16+2+10+6$ $=4+14+6+10$
$=3+13+11+7$ $=15+1+7+11$

Problem Set 28

(i) Find all the 86 ways in which the sum of the elements can be obtained as 34.

(ii) Construct more magic squares with the same 86 patterns.

(iii) Show that you can get $(16\times15\times14\times13)\div24 = 1820$ different groups of four numbers out of the sixteen numbers 1 to 16. Find which of these give interesting geometrical Figures.

Problem 29

In the above magic square, there are patterns in sums of squares and cubes of elements. Find as many of these as possible.

Solution 29

(i) The sum of the elements of the first two rows (columns)
=sum of the elements of the last two rows (columns) = 68.
Sum of the squares of the elements of the first two rows (columns) = sum of the squares of elements of the two last rows (columns = 748).

(ii) Sum of elements of the first and third rows (columns)
= sum of elements of the second and fourth rows (columns) = 68.
Sum of the squares of the elements of the first and third rows (columns).
=sum of the squares of the elements of second and fourth rows (columns) = 768

(iii) Sum of the elements of the diagonals
= sum of the elements not in the diagonals.
Sum of squares of elements in the diagonals.
= sum of squares of elements not in the diagonals.
Sum of the cubes of elements in the diagonals.
=sum of the cubes of elements not in the diagonals.

(iv) $2+8+9+15=3+5+12+14=34$
$2^2+8^2+9^2+15^2=3^2+5^2+12^2+14^2=374$
$2^3+8^3+9^3+15^3=3^3+5^9+12^3+14^3=4624$

Problem Set 29

(i) Double every element of the above square and write all the more than one hundred patterns observed above.

(ii) Add 4 to every element of the above square and find which patterns would survive for the new square.

Problem 30

Take any three digit number, rearrange the digits and subtract the smallest number from the largest number. Repeat the process. What pattern do you observe ? Find similar patterns for 4 and 5 digit numbers.

Solution 30

(a) Suppose we start with 123, we get successively

321	981	972	963	954	954
123	189	279	369	459	459
198	792	693	594	495	495

We can always verify that whatever be three digit number we start with, and go on using the process of rearranging the digits and subtracting the smaller from the largest number, we ultimately end with the number 495 which goes on repeating itself.

(b) For four digit numbers, let us start with 1234 to get

4321	8730	8532	7641
1234	0378	2358	1467
3087	8352	6174	6174

We can verify that whatever the number we start with, we always get the number 6174 which then goes on repeating itself.

(c) For five digit number let us start with 12345 to get

54321	97641	98622	97533	96543	97641
12345	14679	22689	33579	34569	14679
41976	82962	75933	63954	61974	82962

We can verify that whatever the number we start with we always get the cycle of numbers 14679,

82962, 75933, 63954, 14679 and the cycle repeats itself. It may be noted that if we start with any other permutation of 14679, the largest and smallest numbers will be 79641 and 14679 and again we shall get the same cycle.

Problem Set 30

(i) Get the cycles for 6 and 7 digit numbers.

(ii) Show that whatever be the number of digits in the number one starts with, one will end with a cycle of numbers which repeat themselves again and again. Show how you can use a computer to get cycles for numbers with larger number of digits.

Problem 31

Consider the so-called Lucas numbers 1, 3, 4, **7**, 11, 18, 29, 47.....in which any number after the second is the sum of the two preceding numbers. Prove that $b_n = 8\, b_{n-5} + 5b_{n-6}$ and deduce that not a single number of this sequence is divisible by 5.

Solution 31

Since $b_n = b_{n-1} + b_{n-2}$ for all values of n, we get,

$b_n = b_{n-1} + b_{n-2} = 2b_{n-2} + b_{n-3} = 3b_{n-3} + 2b_{n-4}$
$= 5b_{n-4} + 3b_{n-5} = 8b_{n-5} + 5b_{n-6}$

Since $5\, b_{n-6}$ is divisible by 5, if b_{n-5} is not divisible by 5, then b_n will not be divisible by 5. Now

b_1. b_2, b_3, b_4, b_5 are not divisible by 5.

$\therefore$ b_6, b_7, b_8, b_9, b_{10} are not divisible by 5,

$\therefore$ b_{11}, b_{12}, b_{13}, b_{14}, b_{15} are not divisible by 5. Proceeding

in this way, we find that none of the Lucas numbers is divisible by 5.

Problem Set 31

(i) Is any Lucas number divisible by 8 ?

(ii) If $b_n = Ax^n$ prove that $Ax^n = Ax^{n-1} + Ax^{n-2}$

or $x^2 - x - 1 = 0$

or $x = \frac{1 \pm \sqrt{5}}{2}$ so that

$$b_n = A_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + A_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

Determine A_1, A_2 by using $b_1 = 1$, $b_2 = 3$ and verify the formula obtained for b_n for $n = 5, 6, 7, 8$.

Problem 32

Find the unit digit of $(3157)^{1024}$

Solution 32

Unit digit of $(3157)^1$ is 7

Unit digit of $(3157)^2$ = unit digit of $7 \times 7 = 9$

Unit digit of $(3157)^4$ = unit digit of $(9)^2 = 1$

Unit digit of $(3157)^{1024}$ = unit digit of $(1)^{256} = 1$

Problem Set 32

(i) Find the unit digits of $(21571)^{100}$, $(21572)^{1026}$, $(21573)^{1025}$, $(2157)^{57314}$, $(21577)^{517}$.

(ii) Find the number of digits in each of the above numbers.

Problem 33

I have one 'laddoo' of 2" radius. With the same material, how many laddoos of 1" radius can I make ?

Solution 33

The volume of the laddoo $= \frac{4\pi}{3}(2)^3$

$= \frac{32\pi}{3}$ cubic inches

The volume of the new laddoo $= 4/3\pi\,(1)^3$

$= 4/3\pi$ cubic inches.

Volume of one large laddoo = volume of 8 small laddoos.

This verifies that the volume of a sphere varies as the cube of its radius.

Problem Set 33

(i) On a scale map of a country distances are reduced in the ratio 10000 : 1. In what ratios are the areas reduced ?

(ii) Three wooden elephants are available in the market. Their heights are in the ratio 1: 2: 3. Assuming that the dimensions are all changed in the same ratio, show that the volumes of the wood used are in the ratio 1: 8: 27.

(iii) What are the ratios of the surface areas of the elephants ?

Problem 34

Find the first value of n for which n^2+n+41 is not prime.

Solution 34

The numbers we get are

n	1	2	3	4	5	6
n^2+n+41	43	47	53	61	71	83
n	7	8	9	10	11	
n^2+n+41	97	113	131	151	173	

n	12	13	14	15	16	17
n^2+n+41	197	223	251	281	313	347
n	18	19	20	21	22	
n^2+n+41	383	421	461	513	547	
n	23	24	25	26	27	28
n^2+n+41	593	641	691	743	797	853
n	29	30	31	32	33	
n^2+n+41	911	971	1033	1097	1163	
n	34	35	36	37	38	39
n^2+n+41	1231	1301	1373	1417	1523	1601
n	40					
n^2+n+41	1681					

Comparing with the table of primes (extended beyond 1000) in solution 12, we find that the first composite number in the sequence is obtained when $n=40$. A pattern which holds for 39 values breaks down at the 40th value !

No pattern is established simply because it holds 100, 1000 or 10000 times. In mathematics, an observed pattern remains a conjecture until it is established mathematically and rigorously for all values of n.

An effort was made to find a polynomial in n which always represents a prime number. n^2+n+41 was proposed as a candidate. This failed. Now we know that no such polynomial exists.

Problem Set 34

(i) Show that n^2-n+41 gives primes for $n=1$ to 40

(ii) Show that n^2-n+41 can give us an infinity of composite numbers (e.g. for $n=41$ k)

(iii) Show that twenty of the numbers of the sequence

111, 1111, 11111.....are composite, but the number with 23 ones is prime

(iv) What is the first natural number which is divisible by both 25628 and 39062558 ?

Problem 35

The integers 1, 3, 8 and n have the property that the product of any two when added to one yields a perfect square. What is n ?

Solution 35

$1.3+1=4$, $1.8+1=9$, $3.8+1=25$, and 4, 9, 25 are perfect squares.

We want also $1.n+1$, $3.n+1$, $8.n+1$ to be perfect squares.

By trial and error one can find that $n=120$, but this will take some time.

Alternatively let $n+1=a^2$, $3n+1=b^2$, $8n+1=c^2$, so that

$$c^2-1=8(a^2-1),\quad b^2-1=3(a^2-1)$$

or $$c^2=8a^2-7,\qquad b^2=3a^2-2$$

We now try $a=1, 2, 3, 4$..... $a=1$ is all right but this gives $b=1$, $c=1$, $n=0$. Again $a=3$ gives $b=5$, but it does not give c as an integer. $a=4$ gives $c=11$, but then b is not an integer. Only $a=11$ gives $b=19$, $c=31$, $n=120$.

In this way we have reduced the number of trials from 120 to 11.

Problem Set 35

(i) Find m so that integers 1, 2, 8, 120, m are

such that when any two are multiplied and increased by 1, we get a square number.

(ii) Find five sets of numbers a, b, c, d so that $a^2=b^2+c^2+d^2$.

Problem 36

Show that the sum of the areas of circles drawn with the sides of a right-angled triangle as diameters is equal to the area of the circle with the hypotenuse as diameter.

Solution 36

If a, b are the lengths of two sides of a right-angled triangle and c is the length of the hypotenuse, then areas of three circles drawn on the two sides and hypotenuse as diameters are $\pi a^2/4$, $\pi b^2/4$ and $\pi c^2/4$ and the sum of the first two areas=third area, since in a right-angled triangle $a^2+b^2=c^1$.

Problem Set 36

(i) Show that a similar result holds for the areas if we draw semi-circles or equilateral triangles or regular pentagons or hexagons or octagons on the two sides and the hypotenuse.

(ii) What happens if we draw regular polygons of 10 or 12 sides?

(iii) Will the result hold if we draw isosceles triangles with the same base angles?

(iv) Does the result hold if we draw similar (but not necessarily regular) figures on the three sides?

Problem 37

Solve USA+USSR=PEACE i.e. replace the letters

of the alphabet by digits so that this becomes a true equation.

Solution 37

You can verify that the solution is

$$932+9338=10270$$

You may also try to find integers from 0 to 9 which solve the equation

$$100\,U+10\,S+A+1000\,U+100\,S+10\,S+R=10000\,P+1000\,E+100\,A+10\,C+E$$

or $$1100\,U+120\,S+R=10000\,P+1001\,E+99A+10\,C$$

Problem Set 37

Verify the solution of the following problems :

(i) THAT = (AH) × (HA) ; 6786 = (87) × (78)

(ii) (I) × (AM) × (NOT) = (SURE) ;
(1) × (26) × (345) = 8970
(2) × (14) × (307) = 8596

Is he not sure, because there are two answers ?

(iii) HEAT + WATER = STEAM ; No solution is possible.

(iv)

ASHA	9639	9329	9489
GOT	725	846	327
THE	530	620	780
MEDAL	10894	10795	10596

Dr. Sunder Lal has given seven more solutions in Mathematical Education volume 2 No 3, pages 32-38, 1986.

(v) SEND 9567
MORE 1085
MONEY 10652

(vi) Construct some more problems of this type on your own.

Problem 38

Find at least two values of n such that the sum of squares of first n numbers is a perfect square.

Solution 38

One value is obviously I since $1 = 1^2$

The second value is 24 since

$$1^2+2^2+3^2+\ldots\ldots\ldots\ldots+24^2=4900=70^2$$

The sum of the first n natural numbers is $n(n+1)(2n+1)/6$ and so we have to solve the equation $n(n+1)(2n+1)=6m^2$ in integers.

The solutions are $n=1, m=1$; $n=24, m=70$.

Problem Set 38

(i) Find $n(\neq 1)$ so that the sum of the cubes of the first n natural numbers is a perfect square.

(ii) Find n so that the sum of the cubes of the first n natural numbers is a perfect fourth power.

(iii) Can the sum of the first n natural numbers or the sum of their squares or the sum of their cubes be a prime number ?

(iv) Verify that

$18^2+19^2+\ldots\ldots\ldots+28^2=52929=97^2$

$25^3+26^3+\ldots\ldots\ldots\ldots+50^3=38015=195^2$

$456^2+457^2+\ldots\ldots\ldots\ldots+460^2=2357341=1529^2$

$854^2+855^2+\ldots\ldots\ldots\ldots+866^2=8116801=2849^2$

Problem 39

A perfect digital invariant is an integer N of k digits in which the sum of the nth powers of its digits is equal to N. Thus $4151 = 4^5+1^5+5^5+1^5$ ($N=4151$, $k=4$, $n=5$). Find more such numbers.

Solution 39

$467 = 4^3+6^3+7^3$, $153 = 1^3+5^3+3^3$, $370 = 3^3+7^3+0^3$
$371 = 3^3+7^3+1^3$, $407 = 4^3+0^3+7^3$, $548834 = 5^6+4^6+8^6+8^6+3^6+4^6$

Problem Set 39

(i) Find five more such numbers

(ii) Find digits a, b, c, d, varying sum 0 to 9 so that

$abc = a^3+b^3+c^3$, $abc = a^2+b^2+c^2$;

$abcd = a^2+b^2+c^2+d^2$, $abcd = a^3+b^2+c^2+d^2$.

Problem 40

Find all the non-negative integers you can get by using two 2's and usual arithmetic symbols (+, —, ×, division, decimal point, square root, reciprocal).

Solution 40

$2+2=4$, $2-2=0$, $2/2=1$, $2\times2=4$, $2/.2=10$, $(1/.2)^2=25$, $\sqrt{(1/2)}_2=0{\cdot}5$.

Problem Set 40

(i) Find all non-negative numbers you can get by using 2 threes or 3 threes and the arithmetical symbols.

(ii) Find the largest and smallest numbers you can

get by using three threes and the usual arithmetical symbols.

(iii) Which is larger $3^{(3^3)}$ or $(3^3)^3$?

Problem 41

If the length of the edge of a cube is increased by a inches, then the increase in surface area in square inches = the increase in volume in cubic inches. What is the maximum value of a in inches ?

Solution 41

Let the length of the side of the cube be x", so that

$$6[(x+a)^2-x^2]=(x+a)^3-x^3$$

or $3ax^3+x(3a^2-12a)+a^3-6a^2=0$

The roots will be real if $9a^2(a-4)^2 \geqslant 12a^2(a^2-6a)$

or $-a^2+48 \geqslant 0$ or $a^2-48 \leqslant 0$ or $a \leqslant \sqrt{48}$

Problem Set 41

(i) The increase in surface area in square inches of a sphere when its radius is increased by b inches is 1/6th of its increase in cubic inches in volume. What is the maximum value of b ?

(ii) Solve (i) when increase in surface area in square inches = increase in volume in cubic inches.

Problem 42

Express 1/7, 2/7,....., 6/7

1/17, 2/17,....., 16/17

1/29, 2/29,....., 28/29

as recurring decimals and comment on the patterns you get.

Solution 42

$1/7 = \dot{1}42857\dot{}$, $2/7 = .\dot{2}85714\dot{}$, $3/7 = .\dot{4}28571$

$4/7 = .\dot{5}71428\dot{}$, $5/7 = .\dot{7}14285\dot{}$, $6/7 = .\dot{8}57142$

The same six digits occur in the same order in all the six recurring decimals.

$1/17 = 0\dot{5}88235294117647\dot{}$, $2/17 = .\dot{1}176470588235294$

$3/17 = .\dot{1}764705882352941\dot{}$, $4/17 = .\dot{2}352941176470588$

.....

$9/17 = .\dot{5}294117647058823\dot{}$, $11/17 = .\dot{6}47058823529411\dot{7}$

.....

$15/17 = .\dot{8}823529411764705\dot{}$,

$16/17 = \dot{9}411764705882352\dot{}$

The same sixteen digits In the same order occur in all the sixteen recurring decimals.

$1/29 = \dot{0}0344827586206876551724137931\dot{}$

The same 28 digits in the same order occur in all the recurring decimal expressions for 2/29 to 28/29.

Are these not remarkable properties? Will these hold for reciprocals of other prime numbers ?

Problem Set 42

(i) Express the reciprocals of 11, 13, 19, 23 as recurring decimals and note any patterns you may detect.

(ii) Is the reciprocal of every prime number a recurring decimal ?

Problem 43

The numbers 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597.... in which each number is

the sum of the two preceding numbers are called Fibonacci numbers. Let F_n be the nth Fibonacci number. Verify that

$F^2_{n-1}+F^2_n=F_{2n-1}$, $\quad 2F_n-F_{n+1}=F_{n-2}$

$F_1+F_2+\ldots\ldots\ldots+F_n=F_{n+2}-1$

$F^3_{n+1}+F^3_{n+2}-F^3_n=F_m$ for some m. Find more such patterns.

Solution 43

When $n=6$, we have

$F_{n-2}=3$, $F_{n-1}=5$, $F_n=8$, $F_{n+1}=13$, $F_{n+2}=21$, $F_{2n-1}=89$

so that

$F^2_{n-1}+F^2_n=5^2+8^2=89=F_{2n-1}$ ....(1)

$2F_n-F_{n+1}=16-13=3=F_{n-2}$ ....(2)

$\sum_{r=1}^{6} F_r=F_1+F_2+F_3+F_4+F_5+F_6=1+1+2+3+5+8$
$=20=F_{n+1}-1$ ....(3)

$F^3_{n+1}+F^3_{n+2}-F^3_n=13^3+21^3-8^3=10946=F_{21}$ ....(4)

Many of such results can be proved by mathematical induction i.e., we verify the result for $n=1$ and show that if the result is true for any n, then it is also true for $n+1$. Thus to prove the third result, we assume that the result is true for some specified n i.e. that

$F_1+F_2+\ldots\ldots F_n=F_{n+2}-1$

Adding F_{n+1} to both sides we get

$F_1+F_2+\ldots\ldots F_n+F_{n+1}=F_{n+1}+F_{n+2}-1=F_{n+3}-1$

ie. $\sum_{r=1}^{n+1} F_r=F_{n+1}-1$,

so that if the result is true for n it is also true for $n+1$. Also the result is true for $n=1$.

Problem Set 43

(i) Prove the formulae (1) and (2) above.

(ii) Verify the following results for $n=6$, $k=2$, $h=1$:

$F^2_{n+1}-F_nF_{n+2}=(-1)^n$

$F^2_n-F_{n-k}F_{n+k}=(-1)^{n-k}F^2_k$

F_{2n} is divisible by F_n

$F_{n+h}F_{n+k}-F_nF_{n+h+k}=(-1)^nF_hF_k$

(iii) Show that the above results continue to hold for Lucas numbers given in Problem 31.

(iv) Verify the following results :

$F_n+F_{n+2}=L_{n+1}$

$F_nL_n=F_{2n}$

Note : For these and other remarkable results concerning Fibonacci and Lucas numbers see, "A peep into a man-made world made by Fibonacci" in Reflections of a mathematician, by J. N. Kapur, Arya Book Depot, Delhi.

Problem 44

Show that in a Fibonacci sequence an infinite number of members is divisible by 5 and another infinity of members is divisible by 8.

Solution 44

$F_n=F_{n-1}+F_{n-2}=2F_{n-2}+F_{n-3}=3F_{n-3}+2F_{n-4}$

$F=5_{n-4}+3F_{n-5}=8F_{n-5}+5F_{n-6}$

Fibonacci numbers are $F_1=1$, $F_2=1$, $F_3=2$, $F_4=3$, $F_5=5$, $F_6=8$, $F_7=13$, $F_8=21$, so that $F_{10}=8F_5+5F_4$, $F_{15}=8F_{10}+5F_9$, $F_{20}=8F_{15}+5F_{14}$

Since F_5 is divisible by 5, so are

F_{10}, F_{15}, F_{20}.....

As such an infinity of Fibonacci numbers are divisible by 5.

Again $F_{12}=8F_7+5F_6$ $F_{18}=8F_{13}+5F_{12}$

$F_{24}=8E_{19}+5F_{18}$

Since F_6 is divisible by 8, so are F_{12}, F_{18}, F_{24}, F_{30}, F_{36}.....

As such an infinity of Fibonacci numbers are divisible by 8.

Problem Set 44

(i) Show that an infinity of Fibonacci numbers are divisible by 13, 21, 34, 55.

(ii) Show that there is an infinity of Fibonacci numbers which are divisible by any Fibonacci number and their serial numbers in each case form an arithmetic progression.

(iii) Does the property in (ii) hold for Lucas numbers ?

Problem 45

Verify that $1/89 = 0.112358$

13
21
34
55
89
144
233
....

$= 0.11235955040673$

so that, in some sense, F_{11} generates the Fibonacci sequence.

Solution 45

This is an interesting and remarkable property of the eleventh Fibonacci number which can be verified by dividing 1 by 89 and writing the result in the format given in the problem.

Problem Set 45

(i) Generalised Fibonacci numbers are defined by $F_n = F_{n-1} + F_{n-2} + \ldots\ldots\ldots F_{n-k}$
Show that the ordinary Fibonacci numbers correspond to $k = 2$

(ii) When $k = 3$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, Find F_{20}

(iii) When $k = 4$, $F_1 = 1$, $F_2 = 1$, $F_2 = 2$, $F_4 = 4$, find F_{26}

Problem 46

Show that if $N = \ldots\ldots\ldots\ldots a_5a_4a_3a_2a_1$ where a_1's are digits 0 to 9, is divisible by 7, then so is the number

$$N_1 = a_1 + 3a_2 + 2a_3 + 6a_4 + 4a_5 + 5a_6 + a_7 + 3a_8 + 2a + 6a_{10} + 4a_{11} + 5a_{12} \ldots\ldots\ldots$$

The cycle (1, 3, 2, 6, 4, 5) is called a multiplier cycle for divisibility by 7. Find similar multiplier cycles for testing divisibility by 11, 13, 17, 19, 23, 29. By using these cycles, test the divisibility of 215656441 by each of these numbers.

Solution 46

The value of $N = \ldots\ldots\ldots a_8a_7a_6a_5a_4a_3a_2a_1$

$$= a_1 + 10a_2 + 100a_3 + 1000a_4 + 10000a_5 +$$

$= (a_1 + 3a_2 + 2a_3 + 6a_4 + 4a_5 + 5a_6 + a_7 + 3a_8 + 2a_9 + 6a_{10} + 4a_{11} + 5a_{12} + a_{13} \ldots\ldots\ldots) +$ a multiple of 7

$(a_1 + 3a_2 + 2a_3 - a_4 - 3a_5 - 1a_6 + a_7 + 3a_8 + 2a_9 - a_{10} - 3a_{11} - a_{12} \ldots\ldots\ldots +]$ a multiple of 7

Thus if N is divisibie by 7, then so is

$N' = a_1 + 3a_2 + 2a_3 - a_4 - 3a_5 - 2a_6 + \ldots\ldots\ldots$

We call (1, 3, 2, —1, —3, —2) as a multiplier cycle.

N' is much smaller than N and testing its divisibility is easier. If N' is again large, we can again use the multiplier cycle to get an even smaller N" and so on.

Problem Set 46

(i) Verify the multiplier cycles for divisibility of N by n

n	Multiplier cycle
3	(1)
7	(1, 3, 2, —1, —3, —2)
9	(1)
11	(1, —1)
13	(1, —3, —4, —1, 3, 4)
17	(1, —7, —2, —3, 4, 6, —8, 5, —1, 7, 2, 3, —4, —6, 8, —5)
19	(1, —9, 5, —7, 6, 3, —8, —4, —2, —1. 9. —5, 7, —6, —3, 8, 4, 2)
23	(1, 10, 8, 11, —5, —4, 6, —9, 2, —3, —7, —1, —10, —8, —11, 5, 4, —6, 9, —2, 3, 7)

(ii) Verify that

(a) The number of members in the multiplier cycle = number of digits in the recurring expression for 1/N.

(b) When N is prime, the first half of the multiplier cycle differs only in sign from the second half.

(iii) Show that if a number is divisible by 1001, it is divisible by 7, 11, 13. Is the converse true ?

Problem 47

47. Take an equilateral triangle, divide each side into three equal parts and draw an equilateral triangle on the middle part of each side outside the original triangle to get a twelve-sided polygon.
Repeat the process to get a 48-sided polygon.
If the length of sides of original triangle is a, show that the lengths of sides of subsequent polygons are a/3, a/9, a/27,..... and the perimeters of the original triangle and polygons are 3a, 12.a/3, 48.a/9, 192.a/27....., which form a GP with common ratio 4/3. Show that the perimeter approaches infinity as the number of operations increases to infinity. Find also the areas of the polygons and show that the area of polygon approaches a finite limit as the number of operations increases to infinity, so that ultimately we get a polygon with infinite perimeter and finite enclosed area.

Solution 47

[See figures 25—28]

(a) Perimeter

Let a be the length of each side of the original triangle, then perimeter P_1 at the first stage $=3a$.

At the second stage we have 12 sides each of length a/3 so that $P_2=12a/3=4a$

At the third stage we have 48 sides each of length a/9 and then perimeter $P_3=48a/9=16a/3$

Thus the perimeters form a geometric sequence

$$a,\ 4a,\ 16a/3,\ 48a/9,\ 192a/27,.....$$

with common ratio 4/3, which is greater than unity. As such the perimeter becomes larger and larger and appro-

aches infinity as the number of operations becomes larger and larger.

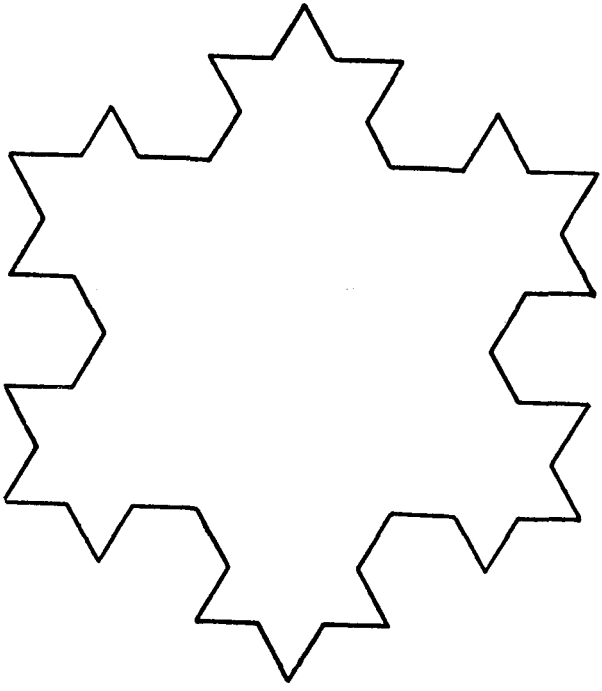

Fig. 25. The third stage C_3.

(b) Area Enclosed

The original triangle is equilateral of side a and its area $= a^2\frac{\sqrt{3}}{4}$

At the second stage, three equilateral triangles of side a/3 are added, so that

$$A_2 = \frac{a^2\sqrt{3}}{4} + 3(a/3)^2\ \sqrt{3}/4$$

At the third stage 12 equilateral triangles of side a/9 are added so that

$$A_3 = \frac{a^2\sqrt{3}}{4} + 3(a/3)^2\ \frac{\sqrt{3}}{4} + 12\ (a/9)^2\ \frac{\sqrt{3}}{4}$$

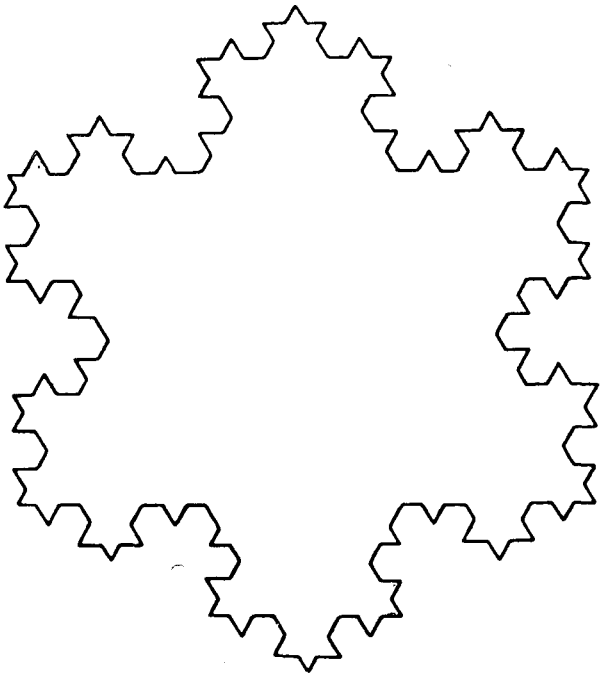

Fig. 26. The fourth stage C_4.

so that the total area enclosed by the ultimate polygon is

$$a^2 \frac{\sqrt{3}}{4}\left(1+\frac{3}{3^2}+\frac{12}{9^2}+\frac{48}{27^2}+\frac{192}{81^2}+\ldots\ldots\right)$$

The series inside the brackets is a geometric series with common ratio 4/9 which is less than unity and as such finite area is enclosed and it is equal to

$$a^2\sqrt{3}/4\ \left(1+\frac{1/3}{1-4/9}\right)$$

$$=a^2\sqrt{3}/4\ (1+3/5)=2\ a^2\sqrt{3}/5$$

which is 8/5 times the area of the original equilateral triangle and is finite.

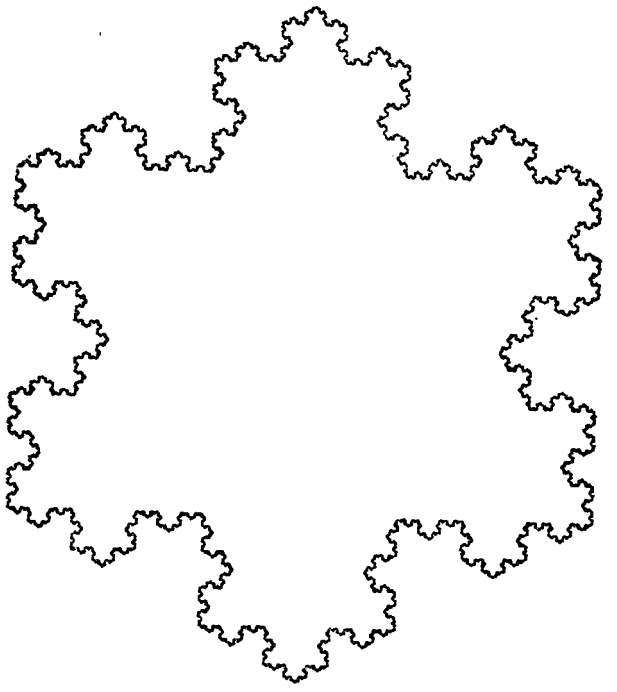

Fig. 27. The fifth stage C_5.

The curve obtained is called the *snow-flake curve.* It has an infinite perimeter and encloses a finite area and has no tangent at any point.

Problem Set 47

(i) Draw the first four stages of the anti snow-flake curve which is drawn in the same way as the snow flake curve; only the new triangles are drawn inward and not outward. [See Figures 29—32]. Show that its perimeter is infinite and area is finite.

(ii) Draw the first four stages of the in-and-out curve as follows: Take six points on a circle so as to divide it into six equal arcs. Turn the

three alternative arcs inward. Next trisect each of the six arcs and turn alternative arc inward

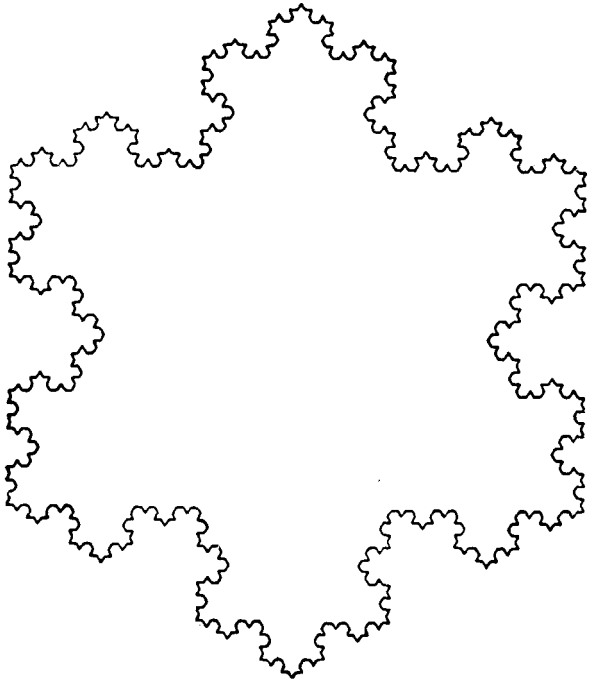

Fig. 28. The sixth stage C_4.

and outward. Repeat the process three times. Show that length of the curve and area enclosed remain constant and finite. [See Figures 33—36].

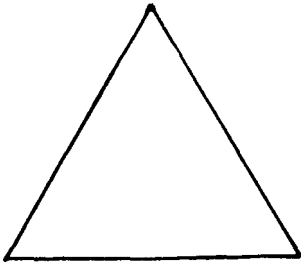

Fig. 29

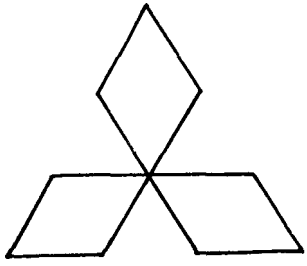

Fig. 30

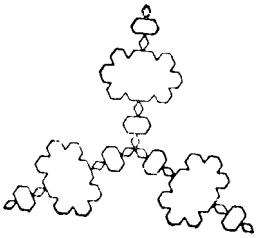

Fig. 31

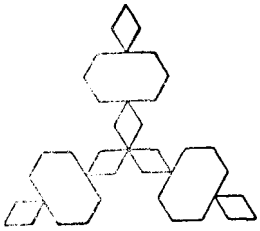

Fig. 32

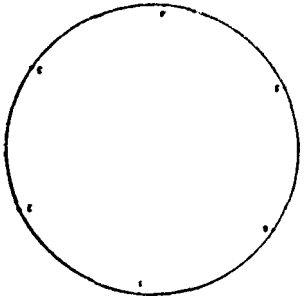

Fig. 33. The in-and-out curve stage C.

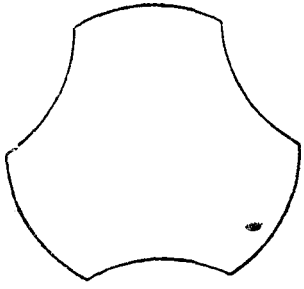

Fig. 34. Stage C_2.

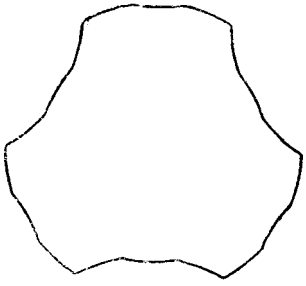

Fig. 35. Stage C_2.

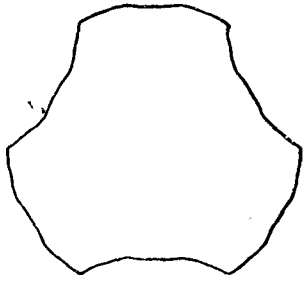

Fig. 36. Stage C_3.

Problem 48

Take a triangle, a quadrilateral, a pentagon, a hexagon, heptagon (all convex). Make 100 copies of each. Try to cover the ground with each. In which cases do you succeed and in which cases do you fail ?

Solution 48

The sum of the exterior angles of a convex polygon is 360°, If the polygon is regular and of n sides each external angle is 360°/n and each internal is

$$\left(180 - \frac{360}{n}\right)^0$$

If m of these polygons meet at a point (common vertex,) we get

$$m(180-360/n)=360 \text{ or } m\,(1-2/n)=2 \qquad (1)$$

Its possible solutions in integers are :

(i) n=3, m=6 which gives us six equilateral triangles meeting at a point giving us tessellation given in figure 37.

(ii) n=4, m=4 which gives us four squares meeting at a point giving us tesselation given in figure 38.

(iii) n=6, m=3 which gives us three hexagons

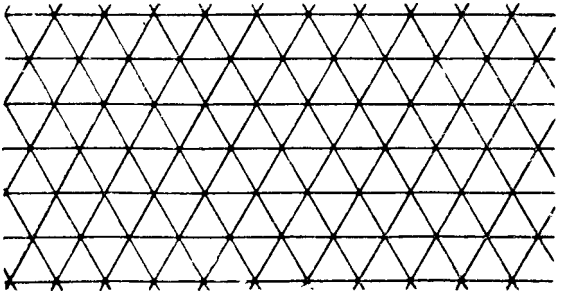

Fig. 37

meeting at a point giving us the tesselation given in figure 39.

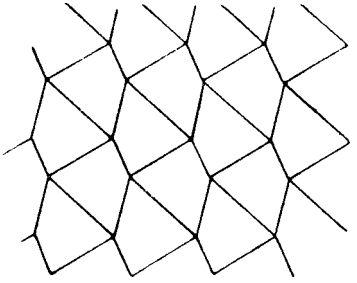

Fig. 38

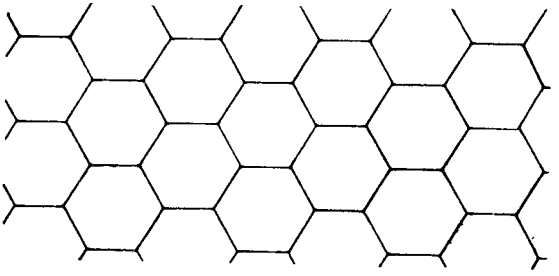

Fig. 39

We can also have a number of polygons of n_1, n_2, n_3 sides meeting at a point given by the equation $(1-2/n_1)+(1-2/n_2)+(1-2/n_3)\ldots\ldots = 2$ (2)

This has 17 solutions in integers, three of which are given above. Some other solutions are :

(iv) $n_1=6$, $n_2=6$, $n_3=3$, $n_4=3$; two triangles and two hexagons meeting at a point (Figure 40).

(v) $n_1=4$, $n_2=8$, $n_3=8$; one square and two octagons meeting at a point (Figure 41).

(vi) $n_1=3$, $n_2=12$, $n_3=12$; one triangle and two twelve sided polygons meeting at a point (Figure 42).

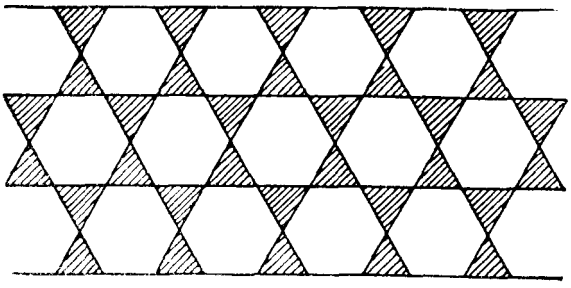

Fig. 40

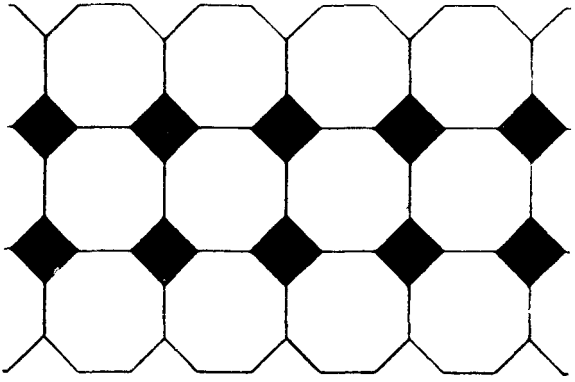

Fig. 41

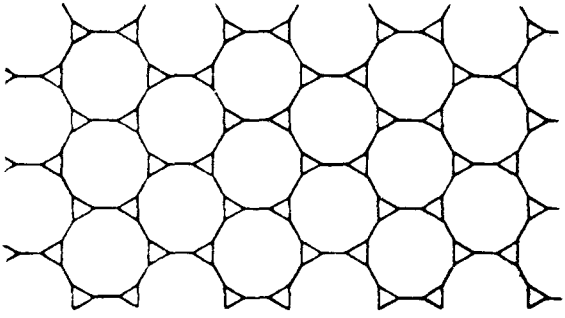

Fig. 42

(vii) $n_1=4$, $n_2=6$, $n_3=12$; one square, one hexagon and one twelve sided polygon meeting at a point (Figure 43)

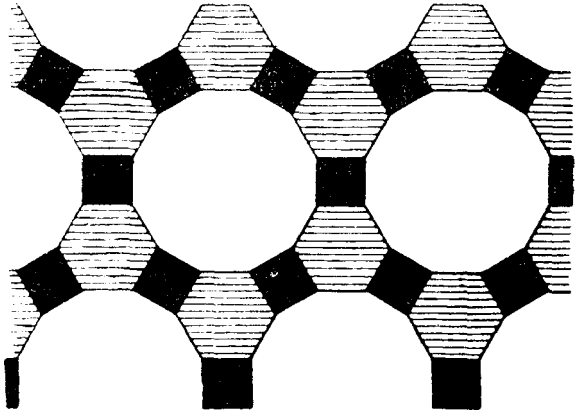

Fig. 43

(viii) $n_1=4$, $n_2=4$, $n_3=3$, $n_4=6$; two squares, one triangle and one hexagon meeting at a point (Figure 44).

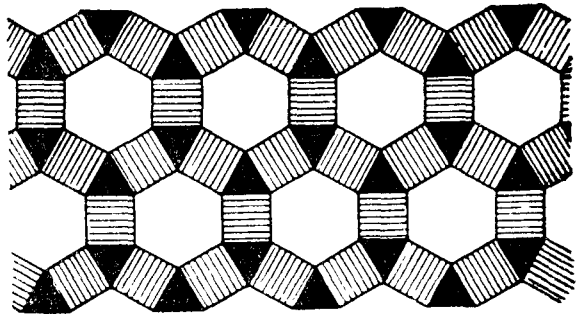

Fig. 44

(ix) $n_1=3$, $n_2=3$, $n_3=3$, $n_4=4$, $n_5=4$; Two squares and three trianges meeting at a point (Figures 45 and 46).

(x) $n_1=3$, $n_2=3$, $n_3=3$, $n_4=3$, $n_5=6$; Four triangles and one hexagon meeting at a point (Figure 48).

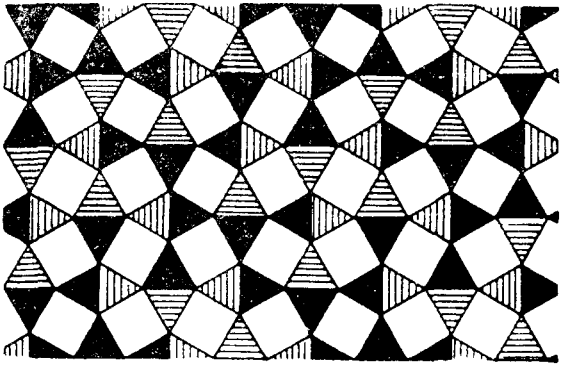

Fig. 45

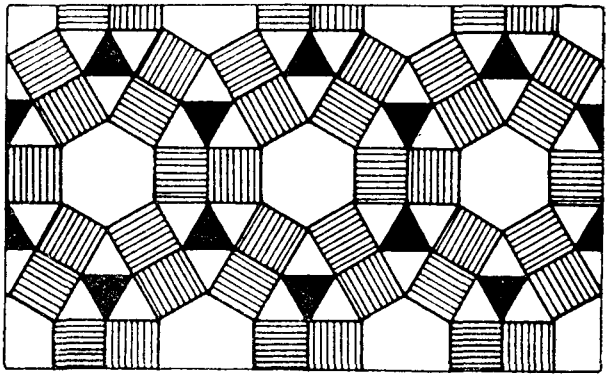

Fig. 46

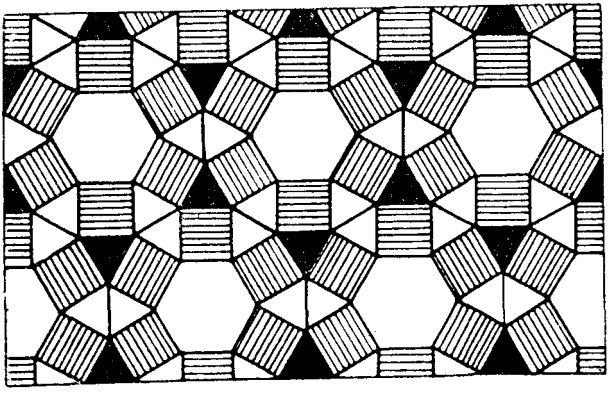

Fig. 47

(xi) $n_2=3$, $n_2=3$, $n_3=3$, $n_4=4$, $n_5 4$: Three triangles and two squares meeting at a point (Figure 47).

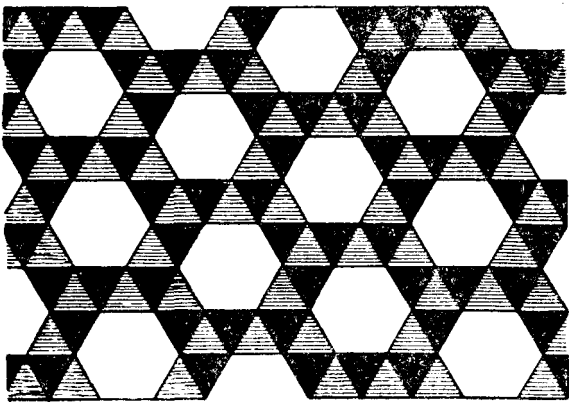

Fig. 48

Problem Set 48

(i) Draw all the eleven tesselations yourself.

(ii) Find some other solutions of (2) and show that the polygons overlap.

Problem 49

(a) Out of the figures 49, 50, 51, 52 given below,

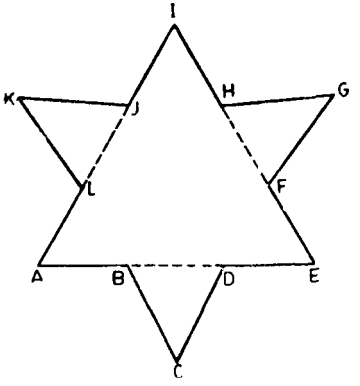

Fig. 49

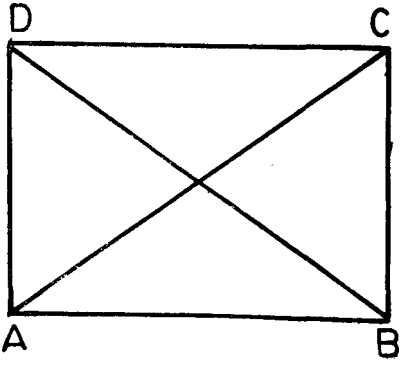

Fig. 50

which onesc an you draw without lifting the pencil from the paper. Try to find a common property of these figures.

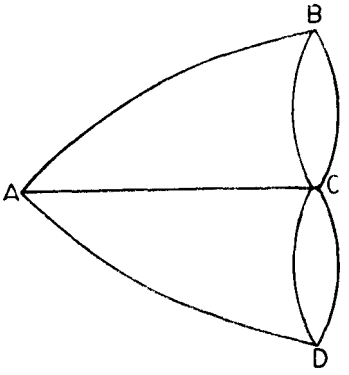

Fig. 51

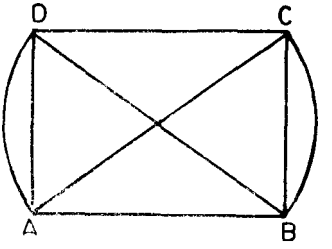

Fig. 52

(b) Can you join A, B, C with each of 1, 2, 3 (Fig. 53) by paths, straight or curved, which do not intersect,

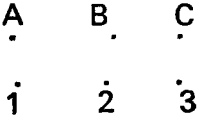

Fig. 53

Solution 49

It is possible to draw figures 49 and 52 without lifting the pencil from the paper.

It is not possible to draw figures 50 and 51 without lifting the pencil from the paper.

Figure 51 corresponds to the famous Konigsberg seven-bridges problem. There are seven bridges shown by the seven arcs : One bridge joining A to B, one bridge joining A to C; one bridge joining A to D, two bridges joining B to C and two bridges join-

ing C to D. A person has to start from any of the vertices A, B, C, D and return to his original position after travelling over all the bridges once and once only. This is equivalent to drawing figures without lifting the pencil from the paper. You can try as much as you like, but you won't succeed. In fact people in Konigsberg did worse; they walked over the actual bridges and failed after a great deal of physical effort.

The number of edges meeting at a vertex is called the degree of the vertex. In figures 49 and 52, the degrees are all even while in figures 49 and 51 all the four vertices are of odd degree. If there are no odd values, the figures can be drawn without lifting the pencil from the paper.

Problem Set 49

(i) In figure 53, three houses A, B, C have to be joined to three utilities (say gas, water, electricity) 1, 2, 3 by pipes which have not to intersect. See if you can draw non-intersecting paths joining each of A, B, C to each of 1, 2, 3.

(ii) Each of four neighbours has connected his house with the other three houses with paths which do not cross. A fifth person builds a house nearby. Prove that he cannot connect his house with all others by non-intersecting paths, but he can connect his house with three of the houses.

Problem 50

There are many methods of sending messages secretly. One method is to replace letters e.g. we can

replace A by C, B by D,....., X by Z, Y by A, Z by B or we can reverse the letters in each word or we can replace letters by certain symbols agreed to in advance. Thus

I LOVE MATHEMATICS can become K NQ X G OCVJGOCVKEU

MATHEMATICS IS FUN can become SCITAMEHTAM SI NUF

Another simple method is to write the message in rectangular form according to columns and send it according to rows. Thus the message

ENEMY PLANS ATTACK AT DAWN is written as

E Y N T A W
N P S A T N
E L A C D Y
M A T K A Z

Here Y, Z are dummy letters. It is coded and sent as follows:

EYNT AWNP SATN ELAC DYMA TKAZ

Coding and decoding are highly interesting mathematical problems. Code the following message according to each of the above methods.

PROBLEM SOLVING IS THE HEART OF MATHEMATICS.

Solution 50

(i) QSPCMN TPMWJOH JT UIF IFBSU PG NBUIFNBUJDT

(ii) MELBORP GNIVLOS SI EHT TRAEH FO SCITAMEHTAM

(iii) PMNE OESR SGHF MAOO IEMA BBLS AATC LVTR TIDE IHTA CEFG

Problem Set 50

Prepare codes for the following messages :

(i) MATHEMATICS IN FUN IS EARNEST AND MATHEMATICS IN EARNEST IS FUN

(ii) MATHEMATICS IS BOTH QUEEN AND HANDMAIDEN OF SCIENCES

(iii) MATHEMATICS IS FOR THOSE WHO LOVE TO CLIMB INTELLECTUAL MOUNTAINS

(iv) MATHEMATICS IS THE MOST BEAUTIFUL ART AND THE MOST USEFUL BASIC SCIENCE.

ERRATA

Page	*Line*	*Instead of*	*read*
16	9b	15	16
22	11t	squares	square
22	13b	p (1) = 2, p (2)=1	p (1)=1, p (2)=2
24	14b	another number	cube number
28	2b	four dimensional	three dimensional
30	7t	than, perimetar	then, perimeter
30	1b	reactangle	rectangle
31	5b	$n\sqrt{x_1x_2.....x_n}$	$\sqrt[n]{x_1x_2.....x_n}$
32	3b	1089089	10891089
36	11b	1x2x3+	1x2x3x
37	3t	log 10^2, 3010	$\log_{10}{}^2$, 3010
41	4b	1/90	1/9 th
50	8b	$2^{11}+-2^8$	$2^{11}+2^8$
52	7b	134	134^4
52	16t	LH	RH
53	8b	24	2^2

NOTE: 11t means 11th line from the top
11b means 11th line from the bottom